GATE Previous Years' Questions

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1. The dispersion relation of a certain wave is **[GATE 1998]**

$$\omega = \sqrt{c^2k^2 + m^2}$$

where ω is the angular frequency, k is the wave vector, c is the velocity of light and m is a constant. The group velocity v of the wave has the following properties

(a) $v \to c$ as $k \to 0$ and $v \to c$ as $k \to \infty$ (b) $v \to c$ as $k \to 0$ and $v \to \infty$ as $k \to \infty$

(c) $v \to 0$ as $k \to 0$ and $v \to \infty$ as $k \to \infty$ (d) $v \to 0$ as $k \to 0$ and $v \to c$ as $k \to \infty$

2. An electron propagating along the x-axis passes through a slit of width $\Delta y = 1\ nm$. The uncertainty in the y-component of its velocity after passing through the slit is **[GATE 2002]**

$\Delta y \Delta P_y \sim \hbar$

$\Delta v_y = \frac{\Delta P_y}{m}$

(a) $7.332 \times 10^5\ m/\sec$ (b) $1.166 \times 10^5\ m/\sec$

(c) $3.436 \times 10^5\ m/\sec$ (d) $2.326 \times 10^5\ m/\sec$

3. A parallel beam of electrons of a given momentum pass through a screen S_1 containing a slit and produces a diffraction pattern on a screen S_2 placed behind it. The width of the central maximum observed on the screen S_2 can be increased by **[GATE 2003]**

(a) decreasing the distance between the screen S_1 and S_2

(b) increasing the width of the slit in screen S_1.

(c) decreasing the momentum of the electrons.

(d) increasing the momentum of the electrons.

→ increase करना है तो momentum Decrease

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4. Given a wave with the dispersion relation $\omega = ck + m$ for $k > 0$ and $m > 0$, which one of the following is true? **[GATE 2003]**

(a) the group velocity is greater than the phase velocity

(b) the group velocity is less than the phase velocity

(c) the group velocity and the phase velocity are equal

(d) there is no definite relation between the group velocity and the phase velocity

5. A particle is confined within a spherical region of radius one femtometer (10^{-15} m). Its momentum can be expected to be about

65

(a) $20\frac{keV}{c}$ (b) $200\frac{keV}{c}$ (c) $200\frac{MeV}{c}$ (d) $\frac{GeV}{c}$ $\to \frac{3}{2}\frac{\pi^2\hbar^2}{mL^2}$

6. A proton is confined to a cubic box, whose sides have length $10^{-12}\ m$. What is the minimum kinetic energy of the proton? The mass of proton is $1.67 \times 10^{-27}\ kg$ and Planck's constant is $6.63 \times 10^{-34}\ Js$. **[GATE 2013]**

(a) $1.1 \times 10^{-17}\ J$ (b) $3.3 \times 10^{-17}\ J$ (c) $9.9 \times 10^{-17}\ J$ (d) $6.6 \times 10^{-17}\ J$

7. The Compton wavelength of a proton is ______ fm. (up to two decimal places). **[GATE 2017]**

$(m_p = 1.67 \times 10^{-27}\text{kg}, h = 6.626 \times 10^{-34}\text{ Js}, e = 1.602 \times 10^{-19}\text{C}, c = 3 \times 10^8\text{ms}^{-1})$

8. Consider a one-dimensional potential well of width 3 nm. Using the uncertainty principle $(\Delta x.\Delta p \geq \hbar/2)$, an estimate of the minimum depth of the well such that it has at least one bound state for an electron is $(m_e = 9.31 \times 10^{-31}kg, h = 6.626 \times 10^{-34}J\ s, e = 1.602 \times 10^{-19}C)$: **[GATE 2017]**

(a) 1 μeV (b) 1 meV (c) 1 eV (d) 1 MeV

$\Delta x \Delta P \sim \hbar$

$\Delta P = \frac{\hbar}{\Delta x}$

ANSWER KEY

1. (d) **2. (b)** **3. (c)** **4. (b)** **5. (c)** **6. (c)**

7. (1.30 to 1.34) **8. (b)**

$K.E = \frac{P^2}{2m} \sim \frac{(\Delta P)^2}{2m}$

CSIR-UGC-NET Previous Years' Questions

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~~1.~~ The energy of a free relativistic particle is $E=\sqrt{|\vec{p}|^2 c^2+m^2c^4}$, where m is its rest mass, $\vec{p}$ is its momentum and c is the speed of light in vacuum. The ratio v_g/v_p of the group velocity v_g of a quantum mechanical wave packet (describing this particle) to the phase velocity v_p is **[CSIR June -2018]**

(a) $|\vec{p}|c/E$ (b) $|\vec{p}|mc^3/E^2$ (c) $|\vec{p}|^2 c^2/E^2$ (d) $|\vec{p}|c/2E$

~~2.~~ 27 The wave number k and the angular frequency ω of a wave are related by the dispersion relation $\omega^2=\alpha k+\beta k^3$, where α and β are positive constants. The wave number for which the phase velocity equals the group velocity, is **[CSIR June -2019]**

(a) $3\sqrt{\frac{\alpha}{\beta}}$ (b) $\sqrt{\frac{\alpha}{\beta}}$ (c) $\frac{1}{2}\sqrt{\frac{\alpha}{\beta}}$ (d) $\frac{1}{3}\sqrt{\frac{\alpha}{\beta}}$

ANSWER KEY

1. (c) **2. (b)**

GATE Previous Years' Questions

1. 40 The eigenfunction for the Hamiltonian of a particle of mass m in a one dimensional potential $V(x)$ is given by $\psi(x) = A\exp(-bx^2/2)$, where A and b are constants. It follows that **[GATE 1998]**

(a) $V(x) =$ constant (b) $V(x) \propto x^3$

(c) $V(x) \propto x^2$ (d) $V(x) \propto 1/x$

72 2. A particle of mass 'm' in one dimension is in a state described by **[GATE 1998]**

$$\psi(x,t) = A\exp\left(\frac{ipx - iEt}{h}\right) + B\exp\left(\frac{-ipx - iEt}{h}\right)$$

The probability current density is given by

(a) $\frac{p}{m}$ (b) $\left(|A| - |B|\right)\frac{p}{m}$ (c) $\left(|A|^2 - |B|^2\right)\frac{p}{m}$ (d) $\left(|A|^2 + |B|^2\right)\frac{p}{m}$

3. Bound state eigenfunctions of an attractive finite range smooth potential behave for larger r as: $r \to \infty, \psi \to 0$ finite **[GATE 2000]**

(a) $\exp(-r/r_0)$ where r_0 is a positive constant. (b) $(1/r^n)$ where n>0

(c) constant (d) $\exp(ikr)/r$ ✗

4. A normalized state of a particle moving in a potential $V(x)$ is given by

$$\psi(x,t) = \sum_{n=0}^{\infty} C_n \phi_n(x) e^{-iE_n t/\hbar}$$

where $\phi_n(x)$'s are the normalized eigenfunctions of the particle corresponding to the energies E_n's. Then **[GATE 2001]**

(a) $\sum_{n=0} |C_n|^2 = 1$

(b) The average energy of the particle in the state $\psi(x,t)$ is $\sum_{n=0}^{\infty} |C_n|^2 E_n$

✗ (c) $\psi(x,t)$ is an eigenfunction of the Hamiltonian of the particle.

✗ (d) $\psi(x,t)$ is an eigenfunction of the momentum operator.

5. Which of the following functions represents acceptable wave function of a particle in the range $-\infty \le x \le \infty$. **[GATE 2001]**

(a) $\varphi(x) = A\tan x, A > 0$ (b) $\varphi(x) = B\cos x$, B real

(c) $\varphi(x) = C\exp(-D/x^2), C > 0, D < 0$ (d) $\varphi(x) = E\,x\exp(-Fx^2), E, F > 0$

6. 127 Formula. An electron in a time independent potential is in a state which is a superposition of the ground state ($E_0 = 11\,eV$) and the first excited state ($E_1 = 13\,eV$). The wave function of the electron will repeat itself with a period of **[GATE 2003]**

(a) 3.1×10^{-18} sec (b) 2.1×10^{-15} sec (c) 1.2×10^{-12} sec (d) 1.0×10^{-9} sec

$$T = \frac{2\pi\hbar}{E_1 - E_0} = \frac{h}{E_1 - E_0} = \frac{6.63\times10^{-34}}{(13-11)\times1.6\times10^{-19}} = 2.1\times10^{-15}\text{ sec.}$$

(b).

456

Commom data for Q. 7 and Q. 8

The normalized wave function ψ_1 and ψ_2 correspond to the ground state and the first excited state of a particle in a potential. You are given the information that the operator $\hat{A}$ acts on the wave functions as $\hat{A}\psi_1 = \psi_2$ and $\hat{A}\psi_2 = \psi_1$ **[GATE 2003]**

7. The expectation value of A for the state $\psi = (3\psi_1 + 4\psi_2)/5$ is :

(a) –0.32 (b) 0.0 (c) 0.75 (d) 0.96

8. Which of the following are eigenfunctions of $\hat{A}^2$?

(a) ψ_1 and ψ_2 (b) ψ_2 and not ψ_1 (c) ψ_1 and not ψ_2 (d) neither ψ_1 and ψ_2

9. 39 The wave function of a spin-less particle of mass m in a one-dimensional potential $V(x)$ is $\psi(x) = A\exp(-\alpha^2x^2)$ corresponding to an eigenvalue $E_0 = \hbar^2\alpha^2/m$. The potential $V(x)$ is **[GATE 2004]**

(a) $2E_0(1-\alpha^2x^2)$ (b) $2E_0(1+\alpha^2x^2)$ (c) $2E_0\alpha^2x^2$ (d) $2E_0(1+2\alpha^2x^2)$

10. 71 A beam of monoenergetic particles having speed v is described by the wave funcion $\psi(x) = u(x)\exp(ikx)$, where $u(x)$ is a real function. This corresponds to a current density **[GATE 2004]**

(a) $u^2(x)v$ (b) v (c) zero (d) $u^2(x)$

11. 41 If the probability that x lies between x and x+dx is $p(x)dx = ae^{-ax}dx$, where $0 < x < \infty$, $a > 0$, then the probability that x lies between x_1 and x_2 $(x_2 > x_1)$ is: **[GATE 2005]**

(a) $(e^{-ax_1} - e^{-ax_2})$ (b) $a(e^{-ax_1} - e^{-ax_2})$ (c) $e^{-ax_2}(e^{-ax_1} - e^{-ax_2})$ (d) $e^{-ax_1}(e^{-ax_1} - e^{-ax_2})$

12. 125 The wavefunction of a particle in a one-dimensional potential at time $t = 0$ is

$$\psi(x, t=0) = \frac{1}{\sqrt{5}}[2\psi_0(x) - \psi_1(x)]$$

where $\psi_0(x)$ and $\psi_1(x)$ are the ground and the first excited states of the particle with corresponding energies E_0 and E_1. The wavefunction of the particle at a time 't' is: **[GATE 2006]**

(a) $\frac{1}{\sqrt{5}}e^{i(E_0E_1)t/2\hbar}[2\psi_0(x) - \psi_1(x)]$ (b) $\frac{1}{\sqrt{5}}e^{-iE_0t/\hbar}[2\psi_0(x) - \psi_1(x)]$

(c) $\frac{1}{\sqrt{5}}e^{-iE_0t/\hbar}[2\psi_0(x) - \psi_1(x)]$ (d) $\frac{1}{\sqrt{5}}[2\psi_0(x)e^{-iE_0t/\hbar} - \psi_1(x)e^{-iE_1t/\hbar}]$

13. 70 आसान A particle of mass m is represented by the wavefunction $\psi(x) = Ae^{ikx}$, where k is the wavevector and A is a constant. The magnitude of the probability current density of the particle is **[GATE 2006]**

(a) $|A|^2\frac{\hbar k}{m}$ (b) $|A|^2\frac{\hbar k}{2m}$ (c) $|A|^2\frac{(\hbar k)^2}{m}$ (d) $|A|^2\frac{(\hbar k)^2}{2m}$

40

~~14.~~ Rough The wave function of a particle, moving in a one-dimensional time-independent potential V(x), is given by $\psi(x)=e^{-iax+b}$, where a and b are constants. This means that the potential $V(x)$ is of the form **[GATE 2007]**

(a) $V(x)\propto x$ (b) $V(x)\propto x^2$ (c) $V(x)=0$ (d) $V(x)\propto e^{-ax}$

235 ✓ both 41 ✓

~~15.~~ For a particle of mass m in a one-dimensional harmonic oscillator potential of the form $V(x)=\frac{1}{2}m\omega^2x^2$, the first excited energy eigen state is $\psi(x)=xe^{-ax^2}$. The value of a is **[GATE 2007]**

(a) $\frac{m\omega}{4\hbar}$ (b) $\frac{m\omega}{3\hbar}$ (c) $\frac{m\omega}{2\hbar}$ (d) $\frac{2m\omega}{3\hbar}$

16 A particle is in the normalized state $|\psi\rangle$ which is superposition of the energy eigenstates $|E_0=10\text{ eV}\rangle$ and $|E_1=30\text{ eV}\rangle$. The average value of energy of the particle in the state $|\psi\rangle$ is 20 eV. The state $|\psi\rangle$ is given by **[GATE 2009]**

(a) $\frac{1}{2}|E_0=10\text{ eV}\rangle+\frac{\sqrt{3}}{4}|E_1=30\text{ eV}\rangle$ (b) $\frac{1}{\sqrt{3}}|E_0=10\text{ eV}\rangle+\sqrt{\frac{2}{3}}|E_1=30\text{ eV}\rangle$

(c) $\frac{1}{2}|E_0=10\text{ eV}\rangle-\frac{\sqrt{3}}{4}|E_1=30\text{ eV}\rangle$ (d) $\frac{1}{\sqrt{2}}|E_0=10\text{ eV}\rangle-\frac{1}{\sqrt{2}}|E_1=30\text{ eV}\rangle$

$\langle E\rangle = P_0E_0 + P_1E_1 = 2$
आना चाहिए

Common Data for Q.17 and Q.18.

A free particle of mass m moves along the x-direction. At $t=0$, the normalized wave function of the particle is given by $\psi(x,0)=\frac{1}{(2\pi\alpha^2)^{1/4}}\exp\left[-\frac{x^2}{4\alpha^2}+i\alpha x\right]$, where α is a real constant **[GATE 2009]**

65

17. The expectation value of the momentum, in this state is

(a) $\hbar\alpha$ (b) $\hbar\sqrt{\alpha}$ (c) α (d) $\frac{\hbar}{\sqrt{\alpha}}$

65

18. The expectation value of the particle energy is

(a) $\frac{\hbar^2}{2m}\frac{1}{2\alpha^{3/2}}$ (b) $\frac{\hbar^2}{2m}\alpha^2$ (c) $\frac{\hbar^2}{2m}\frac{4\alpha^2+1}{4\alpha^{3/2}}$ (d) $\frac{\hbar^2}{8m\alpha^{3/2}}$ (e) $\frac{\hbar^2}{2m}\frac{4\alpha^4+1}{4\alpha^2}$

~~19.~~ 36 Which of the following is an allowed wavefunction for a particle in a bound state? N is a constant and $\alpha,\beta>0$. **[GATE 2010]**

(a) $\psi=N\frac{e^{-\alpha r}}{r^3}$ (b) $\psi=N\left(1-e^{-\alpha r}\right)$

(c) $\psi=Ne^{-\alpha x}e^{-\beta(x^2+y^2+z^2)}$ (d) $\psi=\begin{cases}\text{non-zero constant} & \text{if } r<R\\ 0 & \text{if } r>R\end{cases}$

Common data for Q. 20 and Q. 21

The wavefunction of a particle moving in free space is given by, $\psi=e^{ikx}+2e^{-ikx}$ **[GATE 2012]**

71 ~~20.~~ The energy of the particle is

(a) $\frac{5\hbar^2k^2}{2m}$ (b) $\frac{3\hbar^2k^2}{4m}$ (c) $\frac{\hbar^2k^2}{2m}$ (d) $\frac{\hbar^2k^2}{m}$

71

~~21~~. The probability current density for the real part of the wavefunction is

(a) 1 (b) $\frac{\hbar k}{m}$ (c) $\frac{\hbar k}{2m}$ (d) 0

22. Consider the wave function $Ae^{ikr}(r_0/r)$, where A is the normalization constant. For $r = 2r_0$, the magnitude of probability current density up to two decimal places, in units of $(A^2\hbar k/m)$, is ________.

71

[GATE 2013]

~~23~~. The state of a system is given by

125

[GATE 2016]

$$|\psi\rangle = |\phi_1\rangle + 2|\phi_2\rangle + 3|\phi_3\rangle$$

where, $|\phi_1\rangle, |\phi_2\rangle$ and $|\phi_3\rangle$ form an orthonormal set. The probability of finding the system in the state $|\phi_2\rangle$ is ______________.(Give your answer upto two decimal places)

24. A two-state quantum system has energy eigenvalues $\pm\varepsilon$ corresponding to the normalized states $|\psi_\pm\rangle$. At time $t = 0$, the system is in quantum state $\frac{1}{\sqrt{2}}\left[|\psi_+\rangle + |\psi_-\rangle\right]$. The probability that the system will be in the same state at $t = \frac{h}{(6\varepsilon)}$ is ________ (up to two decimal places).

140

[GATE 2018]

~~25~~. Let $|\psi_1\rangle = \begin{pmatrix}1\\0\end{pmatrix}, |\psi_2\rangle = \begin{pmatrix}0\\1\end{pmatrix}$ represent two possible states of a two-level quantum system. The state obtained by the incoherent superposition of $|\psi_1\rangle$ and $|\psi_2\rangle$ is given by a density matrix that is defined as $\rho \equiv c_1|\psi_2\rangle\langle\psi_1| + c_2|\psi_2\rangle\langle\psi_2|$. If $c_1 = 0.4$ and $c_2 = 0.6$, the matrix element ρ_{22} (rounded off to one decimal place) is ____0.6____

128

$\rho_{22} = \begin{bmatrix} 0 & 0 \\ c_1 & c_2 \end{bmatrix} \to \rho_{22}$

[GATE 2019]

26. The Hamiltonian operator for a two-level quantum system is $H = \begin{pmatrix} E_1 & 0 \\ 0 & E_2 \end{pmatrix}$. If the state of the system at $t = 0$ is given by $|\psi(0)\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}$ then $|\langle\psi(0)|\psi(t)\rangle|^2$ at a later time t is

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[GATE 2019]

(a) $\frac{1}{2}\left(1 + e^{-(E_1-E_2)t/\hbar}\right)$ (b) $\frac{1}{2}\left(1 - e^{-(E_1-E_2)t/\hbar}\right)$

(c) $\frac{1}{2}\left(1 + \cos\left[(E_1 - E_2)t/\hbar\right]\right)$ (d) $\frac{1}{2}\left(1 - \cos\left[(E_1 - E_2)t/\hbar\right]\right)$

ANSWER KEY

1. (c) **2. (c)** **3. (a)** **4. (a & b)** **5. (d)** **6. (b)** **7. (d)**

8. (a) **9. (c)** **10. (a)** **11. (a)** **12. (d)** **13. (a)** **14. (c)**

15. (c) **16. (d)** **17. (a)** **18. (e)** **19. (c)** **20. (c)** **21. (d)**

22. (0.25) **23. (0.27 to 0.29)** **24. (0.25)** **25. (0.6)** **26. (c)**

CSIR-UGC-NET Previous Years' Questions

~~1.~~ 137 The wavefunction of a particle is given by $\psi = \left(\frac{1}{\sqrt{2}}\phi_0 + i\phi_1\right)$, where ϕ_0 and ϕ_1 are the normalized eigenfnctions with energies E_0 and E_1 corresponding to the ground state and first excited state, respectively. The expectation value of the Hamiltonian in the state ψ is: **[CSIR June -2011]**

(a) $\frac{E_0}{2} + E_1$ (b) $\frac{E_0}{2} - E_1$ (c) $\frac{E_0 - 2E_1}{3}$ (d) $\frac{E_0 + 2E_1}{3}$

~~2.~~ 126 The wavefunction of a particle at t = 0 is given by $|\psi(0)\rangle = \frac{1}{\sqrt{2}}\left[|u_1\rangle + |u_2\rangle\right]$, where $|u_1\rangle$ and $|u_2\rangle$ are the normalized eigenstates with eigenvalues E_1 and E_2 respectively, ($E_2 > E_1$). The shortest time after which $|\psi(t)\rangle$ will become orthogonal to $|\psi(0)\rangle$ is **[CSIR Dec 2011]**

(a) $-\frac{\hbar\pi}{(E_2 - E_1)}$ (b) $\frac{\hbar\pi}{(E_2 - E_1)}$ (c) $\frac{\sqrt{2}\hbar\pi}{(E_2 - E_1)}$ (d) $\frac{2\hbar\pi}{(E_2 - E_1)}$

~~3.~~ 50 If a particle is represented by the normalized wave function

$$\psi(x) = \begin{cases} \frac{\sqrt{15}\left(a^2 - x^2\right)}{4a^{5/2}} & \text{for } -a < x < a \\ 0 & \text{otherwise} \end{cases}$$

the uncertainty Δp in its momentum is **[CSIR Dec 2012]**

(a) $\frac{2\hbar}{5a}$ (b) $\frac{5\hbar}{2a}$ (c) $\frac{\sqrt{10}\,\hbar}{a}$ (d) $\frac{\sqrt{5}\,\hbar}{\sqrt{2}\,a}$

~~4.~~ 48 The energies in the ground state and first excited state of a particle of mass $m = \frac{1}{2}$ in a potential V(x) are –4 and –1, respectively, (in units in which $\hbar = 1$). If the corresponding wavefunctions are related by $\psi_1(x) = \psi_0(x)\sinh x$, then the ground state eigenfunction is **[CSIR Dec 2012]**

(a) $\psi_0(x) = \sqrt{\text{sec hx}}$ (b) $\psi_0(x) = \text{sec hx}$ [✓ (c) $\psi_0(x) = \text{sec h}^2\, x$] (d) $\psi_0(x) = \text{sec h}^3\, x$

~~5.~~ 40 If $\psi(x) = A\exp(-x^4)$ is the eigenfunction of a one dimensional Hamiltonian with eigenvalue E = 0, the potential $V(x)$ (in units where $\hbar = 2m = 1$) is **[CSIR Dec 2013]**

(a) $12x^2$ (b) $16x^6$ (c) $16x^6 + 12x^2$ (d) $16x^6 - 12x^2$

~~6.~~ 456 Suppose the Hamiltonian of a conservative system in classical mechanics is $H = \omega xp$, where ω is a constant and x and p are the position and momentum respectively. The corresponding Hamiltonian in quantum mechanics, in the coordinate representation, is **[CSIR Dec 2014]**

(a) $-i\hbar\omega\left(x\frac{\partial}{\partial x} - \frac{1}{2}\right)$ (b) $-i\hbar\omega\left(x\frac{\partial}{\partial x} + \frac{1}{2}\right)$ (c) $-i\hbar\omega x\frac{\partial}{\partial x}$ (d) $-\frac{i\hbar\omega}{2}x\frac{\partial}{\partial x}$

7. 126 Let ψ_1 and ψ_2 denote, the normalized eigenstates of a particle with energy eigenvalues E_1 and E_2 respectively, with $E_2 > E_1$. At time $t = 0$ the particle is prepared in a state $\Psi(t=0) = \frac{1}{\sqrt{2}}(\psi_1 + \psi_2)$. The shortest time T at which $\Psi(t=T)$ will be orthogonal to $\Psi(t=0)$ is **[CSIR Dec 2014]**

(a) $\frac{2\hbar\pi}{(E_2 - E_1)}$ (b) $\frac{\hbar\pi}{(E_2 - E_1)}$ (c) $\frac{\hbar\pi}{2(E_2 - E_1)}$ (d) $\frac{\hbar\pi}{4(E_2 - E_1)}$

8. 127 A Hermitian operator $\hat{O}$ has two normalised eigenstates $|1\rangle$ and $|2\rangle$ with eigenvalues 1 and 2, respectively. The two states $|u\rangle = \cos\theta|1\rangle + \sin\theta|2\rangle$ and $|v\rangle = \cos\phi|1\rangle + \sin\phi|2\rangle$ are such that $\langle v|\hat{O}|v\rangle = 7/4$ and $\langle u|v\rangle = 0$. Which of the following are possible values of θ and ϕ? **[CSIR Dec. 2015]**

(a) $\theta = -\frac{\pi}{6}$ and $\phi = \frac{\pi}{3}$ (b) $\theta = \frac{\pi}{6}$ and $\phi = \frac{\pi}{3}$

(c) $\theta = -\frac{\pi}{4}$ and $\phi = \frac{\pi}{4}$ (d) $\theta = \frac{\pi}{3}$ and $\phi = -\frac{\pi}{6}$

9. 125 The eigenstates corresponding to eigenvalues E_1 and E_2 of a time-independent Hamiltonian are $|1\rangle$ and $|2\rangle$ respectively. If at $t = 0$, the system is in a state $|\psi(t=0)\rangle = \sin\theta|1\rangle + \cos\theta|2\rangle$ the value of $\langle\psi(t)|\psi(t)\rangle$ at time t will be **[CSIR June 2016]**

(a) 1 (b) $\frac{(E_1\sin^2\theta + E_2\cos^2\theta)}{\sqrt{E_1^2 + E_2^2}}$

(c) $e^{iE_1t/\hbar}\sin\theta + e^{iE_2t/\hbar}\cos\theta$ (d) $e^{-iE_1t/\hbar}\sin^2\theta + e^{-iE_2t/\hbar}\cos^2\theta$

10. 50 Consider the two lowest normalized energy eigenfunctions $\psi_0(x)$ and $\psi_1(x)$ of a one dimensional system. They satisfy $\psi_0(x) = \psi_0^*(x)$ and $\psi_1(x) = \alpha\frac{d\psi_0}{dx}$, where α is a real constant. The expectation value of the momentum operator in the state ψ_1 is **[CSIR Dec. 2016]**

(a) $-\frac{\hbar}{\alpha^2}$ (b) 0 (c) $\frac{\hbar}{\alpha^2}$ (d) $\frac{2\hbar}{\alpha^2}$

19 11. 77 The normalized wavefunction in the momentum space of a particle in one dimension is $\phi(p) = \frac{\alpha}{p^2 + \beta^2}$, where α and β are real constants. The uncertainty Δx in measuring its position is **[CSIR Dec. 2017]**

(a) $\sqrt{\pi}\frac{\hbar\alpha}{\beta^2}$ (b) $\sqrt{\pi}\frac{\hbar\alpha}{\beta^3}$ (c) $\frac{\hbar}{\sqrt{2}\beta}$ (d) $\sqrt{\frac{\pi}{\beta}}\frac{\hbar\alpha}{\beta}$

12. 140 The Hamiltonian of a two-level quantum system is $H = \frac{1}{2}\hbar\omega\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$. A possible initial state in which the probability of the system being in that quantum state does not change with time, is **[CSIR Dec. 2017]**

(a) $\begin{pmatrix} \cos\frac{\pi}{4} \\ \sin\frac{\pi}{4} \end{pmatrix}$ (b) $\begin{pmatrix} \cos\frac{\pi}{8} \\ \sin\frac{\pi}{8} \end{pmatrix}$ (c) $\begin{pmatrix} \cos\frac{\pi}{2} \\ \sin\frac{\pi}{2} \end{pmatrix}$ (d) $\begin{pmatrix} \cos\frac{\pi}{6} \\ \sin\frac{\pi}{6} \end{pmatrix}$

13. The wavefunction of a particle of mass m, in a potential $V(x)$ in one dimension is $\psi(x) = Ae^{-\beta|x|}$, where $\beta > 0$ and A are constants. If this wavefunction is an energy eigenfunction, then a possible form of the potential $V(x)$ is **[CSIR Dec. 2019]**

(a) $\frac{\hbar^2}{m}\frac{\beta}{|x|}$ (b) $-\frac{\hbar^2}{m}\frac{\beta}{|x|}$ (c) $\frac{\hbar^2\beta}{m}\delta(x)$ (d) $-\frac{\hbar^2\beta}{m}\delta(x)$

14. 137 Let the normalized eigenstates of the Hamiltonian,

$$H = \begin{pmatrix} 2 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

be $|\psi_1\rangle, |\psi_2\rangle$ and $|\psi_3\rangle$. The expectation value $\langle H\rangle$ and the variance of H in the state $|\psi\rangle = \frac{1}{\sqrt{3}}\left(|\psi_1\rangle + |\psi_2\rangle - i|\psi_3\rangle\right)$ are **[CSIR Dec. 2019]**

(a) $\frac{4}{3}$ and $\frac{1}{3}$ (b) $\frac{4}{3}$ and $\frac{2}{3}$ (c) 2 and $\frac{2}{3}$ (d) 2 and 1

15. 51 The wavefunction of a particle in one-dimension is given to be $\psi(x,t) = u(x)e^{-i\omega t}$, where $u(x)$ is

$$u(x) = \begin{cases} ax & ; \quad 0 \le x \le 1 \\ \frac{1}{2}a(3-x) & ; \quad 1 < x \le 3 \\ 0 & ; \quad \text{otherwise} \end{cases}$$

and $a > 0$. The expectation value of the coordinate $\langle x\rangle$ at time $t = 0$ is **[CSIR Dec. 2019]**

(a) 3/2 (b) 1 (c) 5/4 (d) 3/4

ANSWER KEY

1. (d)	**2. (b)**	**3. (d)**	**4. (c)**	**5. (d)**	**6. (b)**	**7. (b)**
8. (a)	**9. (a)**	**10. (b)**	**11. (c)**	**12. (b)**	**13. (d)**	**14. (c)**
15. (c)						

JEST Previous Years' Questions

~~1.~~ 47 The ground state (apart from normalization) of a particle of unit mass moving in a one-dimensional potential $V(x)$ is $\exp\left(-x^2/2\right)\cosh\left(\sqrt{2}x\right)$. The potential $V(x)$, in suitable units so that $\hbar = 1$, is (up to an additive constant). **[JEST 2012]**

(a) $\frac{x^2}{2}$ (b) $\frac{x^2}{2} - \sqrt{2}x\tanh\left(\sqrt{2}x\right)$

(c) $\frac{x^2}{2} - \sqrt{2}x\tan\left(\sqrt{2}x\right)$ (d) $\frac{x^2}{2} - \sqrt{2}x\coth\left(\sqrt{2}x\right)$

~~2.~~ 114 The wave function of a free particle in one dimension is given by $\psi(x) = A\sin x + B\sin 3x$. Then $\psi(x)$ is an eigenstate of **[JEST 2012]**

(a) the position operator (b) the Hamiltonian

(c) the momentum operator (d) the parity operator

49 ~~3.~~ If the expectation value of the momentum is $\langle p\rangle$ for the wavefunction $\psi(x)$, then the expectation value of momentum for the wavefunction $e^{ikx/\hbar}\psi(x)$ is **[JEST 2013]**

(a) k (b) $\langle p\rangle - k$ (c) $\langle p\rangle + k$ (d) $\langle p\rangle$

~~4.~~ 142 The Hamiltonian operator for a two-state system is given by

$$H = a\left(|1\rangle\langle 1| - |2\rangle\langle 2| + |1\rangle\langle 2| + |2\rangle\langle 1|\right)$$

where a is a positive number with the dimension of energy. The energy eigenstates corresponding to the larger and smaller eigenvalues respectively are **[JEST 2014]**

(a) $|1\rangle - \left(\sqrt{2}+1\right)|2\rangle, |1\rangle + \left(\sqrt{2}-1\right)|2\rangle$ (b) $|1\rangle + \left(\sqrt{2}-1\right)|2\rangle, |1\rangle - \left(\sqrt{2}+1\right)|2\rangle$

(c) $|1\rangle + \left(\sqrt{2}-1\right)|2\rangle, \left(\sqrt{2}+1\right)|1\rangle - |2\rangle$ (d) $|1\rangle - \left(\sqrt{2}-1\right)|2\rangle, \left(\sqrt{2}+1\right)|1\rangle + |2\rangle$

~~5.~~ 141 Consider a three-state system with energies E, E, and E−3g (where g is a constant) and respective eigenstates

$$|\psi_1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\\0\end{pmatrix}, |\psi_2\rangle = \frac{1}{\sqrt{6}}\begin{pmatrix}1\\1\\-2\end{pmatrix}, |\psi_3\rangle = \frac{1}{\sqrt{3}}\begin{pmatrix}1\\1\\1\end{pmatrix}$$

If the system is initially (at t = 0) in state

$$|\psi_i\rangle = \begin{pmatrix}1\\0\\0\end{pmatrix}$$

What is the probability that at a later time 't' system will be in state

$$|\psi_f\rangle = \begin{pmatrix}0\\0\\1\end{pmatrix}$$

[JEST 2014]

(a) 0 (b) $\frac{4}{9}\sin^2\left(\frac{3gt}{2\hbar}\right)$ (c) $\frac{4}{9}\cos^2\left(\frac{3gt}{2\hbar}\right)$ (d) $\frac{4}{9}\sin^2\left(\frac{E-3gt}{2\hbar}\right)$

6. A ball bounces back off earth. You are asked to solve this quantum mechanically assuming the earth is an infinitely hard sphere. Consider surface of earth as the origin implying $V(0)=\infty$ and a linear potential eleswhere (i.e. $V(x)=-mgx \ for \ x>0$). Which of the following wave function is physically admissible for this problem (with $k>0$): **[JEST 2014]**

$\psi = 0$ at $x = 0$ & finite in all space

(a) $\psi(x)=\frac{e^{-kx}}{x}$ (b) $\psi(x)=xe^{-kx^2}$ (c) $\psi(x)=-Axe^{kx}$ (d) $\psi(x)=Ae^{-kx^2}$

7. 136 Given that ψ_1 and ψ_2 are eigenstates of a Hamiltonian with eigenvalues E_1 and E_2 respectively, what is the energy uncertainty in the state $(\psi_1+\psi_2)$? **[JEST 2015]**

(a) $-\sqrt{E_1E_2}$ (b) $\frac{1}{2}|E_1-E_2|$ (c) $\frac{1}{2}|E_1+E_2|$ (d) $\frac{1}{\sqrt{2}}|E_2-E_1|$

8. 98 If a Hamiltonian H is given as $H=|0\rangle\langle 0|-|1\rangle\langle 1|+i(|0\rangle\langle 1|-|1\rangle\langle 0|)$, where $|0\rangle$ and $|1\rangle$ are orthonormal states, the eigenvalues of H are **[JEST 2015]**

(a) ± 1 (b) $\pm i$ (c) $\pm\sqrt{2}$ (d) $\pm i\sqrt{2}$

9. 66 What is the dimension of $\frac{\hbar}{i}\frac{\partial\psi}{\partial x}$, where ψ is a wavefunction in two dimensions? **[JEST 2017]**

(a) $\mathrm{kg\,m^{-1}s^{-2}}$ (b) $\mathrm{kg\,s^{-2}}$ (c) $\mathrm{kg\,m^2s^{-2}}$ (d) $\mathrm{kg\,s^{-1}}$

10. 455 Consider the Hamiltonian

$$H(t)=\alpha\begin{pmatrix}1&0&0\\0&2&0\\0&0&3\end{pmatrix}+\beta(t)\begin{pmatrix}0&0&1\\0&0&0\\1&0&-2\end{pmatrix}$$

The time dependent function $\beta(t)=\alpha$ for $t\le 0$ and zero for $t>0$. Find $|\langle\Psi(t<0)|\Psi(t>0)\rangle|^2$, where $|\Psi(t<0)\rangle$ is the normalized ground state of the system at a time $t<0$ and $|\Psi(t>0)\rangle$ is the state of the system at $t>0$. **[JEST 2017]**

(a) $\frac{1}{2}(1+\cos(2\alpha t))$ (b) $\frac{1}{2}(1+\cos(\alpha t))$ (c) $\frac{1}{2}(1+\sin(2\alpha t))$ (d) $\frac{1}{2}(1+\sin(\alpha t))$

Ques.11 $10000\times P(t=h/6E)=2500$ (Page-14 Ques 24)

ANSWER KEY

1. (b) 2. (d) 3. (c) 4. (b) 5. (b) 6. (b) 7. (b)
8. (c) 9. (d) 10. (a)

TIFR Previous Years' Questions

1. In a quantum mechanical system, an observable A is represented by an operator $\hat{A}$. If $|\psi\rangle$ is a state of the system, but not an eigenstate of $\hat{A}$, then the quantity

$$r = \langle\psi|\hat{A}|\psi\rangle^2 - \langle\psi|\hat{A}^2|\psi\rangle = \langle A\rangle^2 - \langle A^2\rangle = -(\Delta A)^2$$

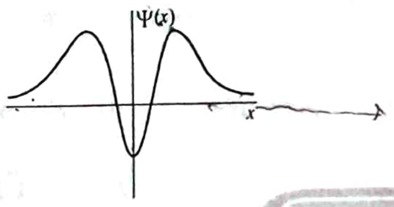
$\Delta A \geq 0$ $\Delta A > 0$

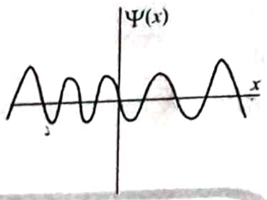
[TIFR 2013]

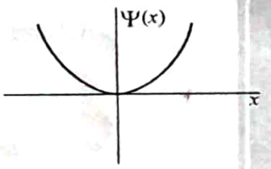
satisfies the relation

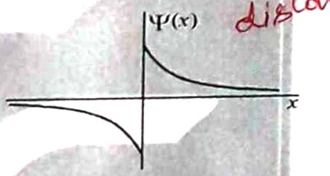
(a) $r < 0$ (b) $r = 0$ (c) $r > 0$ (d) $r \geq 0$

2/136 A particle of energy E moves in one dimension under the influence of a potentials $V(x)$. If $E > V(x)$ for some range of x, which of the following graphs can represent a bound state wave function of the particle? [TIFR 2013]

(a)


(b)


(c)


(d)


discontinuous

3/138 The state $|\psi\rangle$ of a quantum mechanical system, in a certain basis, is represented by the column vector

$$|\psi\rangle = \begin{pmatrix} 1/\sqrt{2} \\ 0 \\ 1/\sqrt{2} \end{pmatrix}.$$

The operator $\hat{A}$ corresponding to a dynamical variable A, is given, in the same basis, by the matrix

$$\hat{A} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix}.$$

If, now, a measurement of the variable A is made on the system in the state $|\psi\rangle$, the probability that the result will be +1 is (Non-deg. हो तो eigen values SKIP करना) [TIFR 2013]

(a) $\frac{1}{\sqrt{2}}$ (b) 1 (c) $\frac{1}{2}$ (d) $\frac{1}{4}$

4. 50 A particle moving in a one dimension has the un-normalised wave function

$$\psi(x) = x\exp\left(-\frac{x^2}{\lambda^2}\right)$$

where λ is a real constant. The expectation value of its momentum is **[TIFR 2014]**

(a) $\frac{\hbar}{\lambda}\exp\left(-\frac{x^2}{\lambda^2}\right)$ (b) $\frac{\hbar^2}{\lambda^2} - 2\frac{\hbar}{\lambda}$ (c) $\frac{\hbar}{\lambda}\exp(-1)$ (d) zero

5. 57 The probability function for a variable x which assumes only positive values is **[TIFR 2014]**

$$f(x) = x\exp\left(-\frac{x}{\lambda}\right)$$

where $\lambda > 0$. The ratio $\langle x\rangle / \hat{x}$, where $\hat{x}$ is the most probable value and $\langle x\rangle$ is the mean value of the variable x, is

(a) 2 (b) $\frac{1+\lambda}{1-\lambda}$ (c) $\frac{1}{\lambda}$ (d) 1

6. 143 A two state quantum system has two observables A and B. It is known that the observable A has eigenstates $|\alpha_1\rangle$ and $|\alpha_2\rangle$ with eigenvalues a_1 and a_2 respectively, while B has eigenstates $|\beta_1\rangle$ and $|\beta_2\rangle$ with eigenvalues b_1 and b_2 respectively, and that these eigenstates are related by **[TIFR 2015]**

$$|\beta_1\rangle = \frac{3}{5}|\alpha_1\rangle - \frac{4}{5}|\alpha_2\rangle \qquad |\beta_2\rangle = \frac{4}{5}|\alpha_1\rangle + \frac{3}{5}|\alpha_2\rangle$$

Suppose a measurement is made of the observable A and a value a_1 is obtained. If the observable B is now measured, the probability of obtaining the value b_1 will be

(a) 0.80 (b) 0.64 (c) 0.60 (d) 0.36

7. 58 If x is a continuous variable which is uniformly distributed over the real line from $x = 0$ to $x \to \infty$ according to the distribution $f(x) = \exp(-4x)$ then the exception value of $\cos 4x$ is **[TIFR 2016]**

(a) zero (b) 1/2 (c) 1/4 (d) 1/16

8. 79 A particle moving in one dimension is confined inside a rigid box located between $x = -a/2$ and $x = a/2$. If the particle is in its ground state $\psi(0) = \sqrt{2/a}\cos\frac{\pi x}{a}$ the quantum mechanical probability of its having a momentum p is given by **[TIFR 2016]**

(a) $\frac{8\hbar^4}{(\pi^2\hbar^2 - p^2a^2)^2}\cos^2\frac{pa}{2\hbar}$ (b) $\frac{\pi^2\hbar^4}{(\pi^2\hbar^2 - p^2a^2)^2}\sin^2\frac{pa}{2\hbar}$

(c) $\frac{2\hbar^4}{(\pi^2\hbar^2 + p^2a^2)^2}\cos^2\frac{pa}{2\hbar}$ (d) $\frac{16\hbar^4}{(\pi^2\hbar^2 - p^2a^2)^2}$

~~9.~~ 485 The **normalized** wave function of a particle can be written as $\Psi(x) = N\sum_{n=0}^{\infty}\left(\frac{1}{\sqrt{7}}\right)^n \varphi_n(x)$, where $\varphi_n(x)$ are the normalized energy eigenfunctions of a given Hamiltonian. The value of N is **[TIFR 2017]**

(a) $\sqrt{\frac{1}{7}}$ (b) $\sqrt{\frac{6}{7}}$ (c) $\sqrt{\frac{3}{7}}$ (d) $\sqrt{\frac{(6-2\sqrt{7})}{7}}$

~~10.~~ 455 A particle of mass m, moving in one dimension, satisfies the modified Schrödinger Equation:

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + i\hbar u\frac{d\psi}{dx} = i\hbar\frac{d\psi}{dt}$$

where u is the velocity of the substrate. If, now, this particle is treated as a Gaussian wave packet peaked at wavenumber k, its group velocity will be $v_g =$ **[TIFR 2019]**

(a) $\frac{\hbar k}{m}+u$ (b) $\frac{\hbar k}{2m}-u$ (c) $-\frac{\hbar k}{2m}+u$ (d) $\frac{\hbar k}{m}-u$

~~11.~~ 455 A beam of high energy neutrons is scattered from a metal lattice, where the spacing between nuclei is around 0.4 nm. In order to see quantum diffraction effects, the energy of the neutrons must be around

(a) 78.5 eV (b) 5.11 meV (c) 511 keV (d) 7.85 MeV **[TIFR 2019]**

ANSWER KEY

1. (a) **2. (a)** **3. (d)** **4. (d)** **5. (a)** **6. (d)** **7. (b)**

8. (a) **9. (b)** **10. (d)** **11. (b)**

GATE Previous Years' Questions

1. 454 x and p are two operators which satisfy $[x, p] = i$. the operators X and P are defined as $X = x\cos\phi + p\sin\phi$ and $Y = -x\sin\phi + p\cos\phi$ for ϕ real. Then [X, Y] equals **[GATE 2001]**
(a) 1 (b) −1 (c) i (d) −i

2. 109 $\hat{A}$ and $\hat{B}$ are two quantum mechanical operators. If $\left[\hat{A}, \hat{B}\right]$ stands for the commutator of $\hat{A}$ and $\hat{B}$, then $\left[\left[\hat{A}, \hat{B}\right], \left[\hat{B}, \hat{A}\right]\right]$ is equal to **[GATE 2002]**
(a) $\hat{A}\hat{B}\hat{A}\hat{B} - \hat{B}\hat{A}\hat{B}\hat{A}$ (b) $\hat{A}\left(\hat{A}\hat{B} - \hat{B}\hat{A}\right) - \hat{B}\left(\hat{B}\hat{A} - \hat{A}\hat{B}\right)$ (c) zero (d) $\left(\left[\hat{A}, \hat{B}\right]\right)^2$

3. For a physical system, two obervables O_1 and O_2 are known to be compatible. Choose the correct implication from amongst those given below **[GATE 2008]**
$\hookrightarrow [O_1, O_2] = 0$
(a) every eigenstate of O_1 must necessarily be an eigenstate O_2
(b) every non-degenerate eigenstate of O_1 must necessarily be an eigenstate of O_2
(c) when an observation of O_1 is carried out on an arbitrary state $|\psi\rangle$ of the physical system, a subsequent observation of O_2 leads to an unambiguous result.
(d) observation of O_1 and O_2, carried out on an arbitrary state $|\psi\rangle$ of the physical system, lead to the identical results irrespective of the order in which the observations are made.

4. 99 Which of the following operators is Hermitian? **[GATE 2016]**
(a) $\frac{d}{dx}$ (b) $\frac{d^2}{dx^2}$ (c) $i\frac{d^2}{dx^2}$ (d) $\frac{d^3}{dx^3}$

5. 108 If x and p are the x components of the position and the momentum operators of a particle respectively, the commutator $\left[x^2, p^2\right]$ is **[GATE 2016]**
(a) $i\hbar(xp - px)$ (b) $2i\hbar(xp - px)$ (c) $i\hbar(xp + px)$ (d) $2i\hbar(xp + px)$

6. 114 For the parity operator P, which of the following statements is NOT true? **[GATE 2016]**
(a) $P^\dagger = P$ (b) $P^2 = -P$ (c) $P^2 = I$ (d) $P^\dagger = P^{-1}$

7. 99. Which one of the following operators is Hermitian? **[GATE 2017]**
(a) $i\frac{(p_x x^2 - x^2 p_x)}{2}$ (b) $i\frac{(p_x x^2 + x^2 p_x)}{2}$ (c) $e^{ip_x a}$ (d) $e^{-ip_x a}$

8. 109 If H is the Hamiltonian for a free particle with mass m, the commutator $[x, [x, H]]$ is **[GATE 2018]**
(a) $\frac{\hbar^2}{m}$ (b) $-\frac{\hbar^2}{m}$ (c) $-\frac{\hbar^2}{(2m)}$ (d) $\frac{\hbar^2}{(2m)}$

ANSWER KEY

1. (c) **2. (c)** **3. (a)** **4. (b)** **5. (d)** **6. (b)** **7. (a)**
8. (b)

CSIR-UGC-NET Previous Years' Questions

[CSIR June 2012]

1. 108 The commutator $\left[x^2, p^2\right]$ is:

(a) $2i\hbar\,xp$ (b) $2i\hbar(xp+px)$ (c) $2i\hbar px$ (d) $2i\hbar(xp-px)$

2. 100 without [illegible] Which of the following is a self-adjoint operator in the spherical polar coordinate system (r,θ,φ)?

[CSIR June 2012]

(a) $-\dfrac{i\hbar}{\sin^2\theta}\dfrac{\partial}{\partial\theta}$ (b) $-i\hbar\dfrac{\partial}{\partial\theta}$ (c) $-\dfrac{i\hbar}{\sin\theta}\dfrac{\partial}{\partial\theta}$ (d) $-i\hbar\sin\theta\dfrac{\partial}{\partial\theta}$

3. 111 Given the usual canonical commutation relations, the commutator $[A,B]$ of $A=i\left(xp_y-yp_x\right)$ and $B=\left(yp_z+zp_y\right)$ is:

[CSIR Dec 2012]

(a) $\hbar\left(xp_z-p_xz\right)$ (b) $-\hbar\left(xp_z-p_xz\right)$ (c) $\hbar\left(xp_z+p_xz\right)$ (d) $-\hbar\left(xp_z+p_xz\right)$

4. 110 If the operators A and B satisfy the commutation relation [A, B] = I, where I is the identity operator, then

[CSIR June 2013]

(a) $[e^A, B] = e^A$ (b) $[e^A, B] = [e^B, A]$

(c) $[e^A, B] = [e^{-B}, A]$ (d) $[e^A, B] = 1$

5. 110 If A, B and C are non-zero Hermitian operators, which of the following relations must be false?

[CSIR Dec 2013]

(a) [A, B] = C (b) AB + BA = C (c) ABA = C (d) A+ B = C

6. 110 Let x and p denote, respectively, the coordinate and momentum operators satisfying the canonical commutation relation $[x,p]=i$ in natural units $(\hbar=1)$. Then the commutator $[x, pe^{-p}]$ is

[CSIR Dec 2014]

(a) $i(1-p)e^{-p}$ (b) $i(1-p^2)e^{-p}$ (c) $i(1-e^{-p})$ (d) ipe^{-p}

7. 162 The waveform of a particle in one-dimension is denoted by $\psi(x)$ in the coordinate representation and by $\phi(p)=\int\psi(x)e^{-ipx/\hbar}dx$ in the momentum representation. If the action of an operator $\hat{T}$ on $\psi(x)$ is given by $\hat{T}\psi(x)=\psi(x+a)$, where a is a constant, then $\hat{T}\phi(p)$ is given by

[CSIR June 2015]

(a) $-\dfrac{i}{\hbar}ap\phi(p)$ (b) $e^{-iap/\hbar}\phi(p)$ (c) $e^{+iap/\hbar}\phi(p)$ (d) $\left(1+\dfrac{i}{\hbar}ap\right)\phi(p)$

8. 145 A particle moves in one dimension in the potential $V=\dfrac{1}{2}k(t)x^2$, where $k(t)$ is a time dependent parameter. Then $\dfrac{d}{dt}\langle V\rangle$, the rate of change of the expectation value $\langle V\rangle$ of the potential energy, is

[CSIR June 2015]

(a) $\dfrac{1}{2}\dfrac{dk}{dt}\langle x^2\rangle+\dfrac{k}{2m}\langle xp+px\rangle$ (b) $\dfrac{1}{2}\dfrac{dk}{dt}\langle x^2\rangle+\dfrac{1}{2m}\langle p^2\rangle$

(c) $\dfrac{k}{2m}\langle xp+px\rangle$ (d) $\dfrac{1}{2}\dfrac{dk}{dt}\langle x^2\rangle$

~~9.~~ 116 Two different sets of orthogonal basis vectors $\left\{\begin{pmatrix}1\\0\end{pmatrix},\begin{pmatrix}0\\1\end{pmatrix}\right\}$ and $\left\{\frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix},\frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix}\right\}$ are given for a two-dimensional real vector space. The matrix representation of a linear operator $\hat{A}$ in these bases are related by a unitary transformation. The unitary matrix may be chosen to be **[CSIR June 2015]**

(a) $\begin{pmatrix}0 & -1\\1 & 0\end{pmatrix}$ (b) $\begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}$ (c) $\frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\1 & -1\end{pmatrix}$ (d) $\frac{1}{\sqrt{2}}\begin{pmatrix}1 & 0\\1 & 1\end{pmatrix}$

~~10.~~ 109 Consider the operator $a = x + \frac{d}{dx}$ acting on smooth functions of x. The commutator $[a, \cos x]$ is **[CSIR Dec. 2016]**

(a) $-\sin x$ (b) $\cos x$ (c) $-\cos x$ (d) 0

~~11.~~ 454 Consider the operator $\vec{\pi} = \vec{p} - q\vec{A}$, where $\vec{p}$ is the momentum operator, $\vec{A} = (A_x, A_y, A_z)$ is the vector potential and q denotes the electric charge. If $\vec{B} = (B_x, B_y, B_z)$ denotes the magnetic field, the z-component of the vector operator $\vec{\pi}\times\vec{\pi}$ is **[CSIR Dec. 2016]**

(a) $iq\hbar B_z + q(A_x p_y - A_y p_x)$ (b) $-iq\hbar B_z - q(A_x p_y - A_y p_x)$

(c) $-iq\hbar B_z$ (d) $iq\hbar B_z$

~~12.~~ 125 The two vectors $\begin{pmatrix}a\\0\end{pmatrix}$ and $\begin{pmatrix}b\\c\end{pmatrix}$ are orthogonal if **[CSIR June 2017]**

(a) $a=\pm1,\ b=\pm1/\sqrt{2},\ c=\pm1/\sqrt{2}$ (b) $a=\pm1,\ b=\pm1,\ c=0$

(c) $a=\pm1,\ b=0,\ c=\pm1$ (d) $a=\pm1,\ b=\pm1/2,\ c=1/2$

~~13.~~ 110 111 Let x denote the position operator and p the canonically conjugate momentum operator of a particle. The commutator **[CSIR Dec. 2017]**

$$\left[\frac{1}{2m}p^2+\beta x^2,\ \frac{1}{m}p^2+\gamma x^2\right],$$

where β and γ are constants, is zero if

(a) $\gamma=\beta$ (b) $\gamma=2\beta$ (c) $\gamma=\sqrt{2}\beta$ (d) $2\gamma=\beta$

~~14~~ 116 The operator A has a matrix representation $\begin{pmatrix}2 & 1\\1 & 2\end{pmatrix}$ in the basis spanned by $\begin{pmatrix}1\\0\end{pmatrix}$ and $\begin{pmatrix}0\\1\end{pmatrix}$. In another basis spanned by $\frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}$ and $\frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix}$, the matrix representation of A is **[CSIR June 2019]**

(a) $\begin{pmatrix}2 & 0\\0 & 2\end{pmatrix}$ (b) $\begin{pmatrix}3 & 0\\0 & 1\end{pmatrix}$ (c) $\begin{pmatrix}3 & 1\\0 & 1\end{pmatrix}$ (d) $\begin{pmatrix}3 & 0\\1 & 1\end{pmatrix}$

~~15.~~ 453 The operator $x\frac{d}{dx}\delta(x)$, where $\delta(x)$ is the Dirac delta function, acts on the space of real-valued square-integrable functions on the real line. This operator is equivalent to **[CSIR June 2019]**

(a) $-\delta(x)$ (b) $\delta(x)$ (c) x (d) 0

16. 453 Let $\hat{x}$ and $\hat{p}$ denote position and momentum operators obeying the commutation relation $[\hat{x}, \hat{p}] = i\hbar$. If $|x\rangle$ denotes an eigenstate of $\hat{x}$ corresponding to the eigenvalue x, then $e^{ia\hat{p}/\hbar}|x\rangle$ is **[CSIR Dec. 2019]**

(a) an eigenstate of $\hat{x}$ corresponding to the eigenvalue x.
(b) an eigenstate of $\hat{x}$ corresponding to the eigenvalue $(x + a)$.
(c) an eigenstate of $\hat{x}$ corresponding to the eigenvalue $(x - a)$.
(d) not an eigenstate of $\hat{x}$.

ANSWER KEY

1. (b) **2. (c)** **3. (c)** **4. (a)** **5. (a)** **6. (a)** **7. (c)**
8. (a) **9. (c)** **10. (a)** **11. (d)** **12. (c)** **13. (b)** **14. (b)**
15. (a) **16. (c)**

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JEST Previous Years' Questions

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1. 100 Let $N = b^{\dagger}b$, where the operators b and $b^{\dagger}$. satisfy the relations $bb^{\dagger} + b^{\dagger}b = 1$ and $b^2 = 0 = (b^{\dagger})^2$. The eigenvalues of N are

(a) Positive and negative integers (b) all positive integers

(c) ±1 and 0 only (d) 0 and 1 only

[JEST 2013]

2. 102 The hermitian conjugate of the operator $(-\partial/\partial x)$ is

(a) $\partial/\partial x$ (b) $-\partial/\partial x$ (c) $i\partial/\partial x$ (d) $-i\partial/\partial x$

[JEST 2013]

3. 96 Similar The operator $\left(\frac{d}{dx} \underset{+}{-} x\right)\left(\frac{d}{dx} \underset{-}{+} x\right)$ is equivalent to

(a) $\frac{d^2}{dx^2} - x^2$ (b) $\frac{d^2}{dx^2} - x^2 + 1$

(c) $\frac{d^2}{dx^2} - x\frac{d}{dx} + 1$ (d) $\frac{d^2}{dx^2} - 2x\frac{d}{dx} - x^2$

4. 110 The operator A and B share all the eigenstates. Then the least possible value of the product of uncertainties $\Delta A \Delta B$ is **[JEST 2014]**

(a) $\hbar$ (b) 0 (c) $\hbar/2$ (d) Determinant (AB)

5. 100 The adjoint of a differential operator $\frac{d}{dx}$ acting on a wavefunction $\psi(x)$ for a quantum mechanical system is:

(a) $\frac{d}{dx}$ (b) $-i\hbar\frac{d}{dx}$ (c) $-\frac{d}{dx}$ (d) $i\hbar\frac{d}{dx}$ **[JEST 2016]**

[JEST 2016]

6. 452 For operators P and Q, the commutator $[P, Q^{-1}]$ is:

(a) $Q^{-1}[P,Q]Q^{-1}$ (b) $-Q^{-1}[P,Q]Q^{-1}$ (c) $Q^{-1}[P,Q]Q$ (d) $-Q^{-1}[P,Q]Q^{-1}$

7. 453 Let $\Lambda = \begin{pmatrix} 1 & 0 \\ 0 & 11 \end{pmatrix}$ and $M = \begin{pmatrix} 10 & 3i \\ -3i & 2 \end{pmatrix}$. Similarly transformation of M to Λ can be performed by

[JEST 2017]

(a) $\frac{1}{\sqrt{10}}\begin{pmatrix} 1 & 3i \\ 3i & 1 \end{pmatrix}$ (b) $\frac{1}{\sqrt{9}}\begin{pmatrix} 1 & -3i \\ 3i & 11 \end{pmatrix}$ (c) $\frac{1}{\sqrt{10}}\begin{pmatrix} 1 & 3i \\ -3i & 11 \end{pmatrix}$ (d) $\frac{1}{\sqrt{9}}\begin{pmatrix} 1 & 3i \\ -3i & 1 \end{pmatrix}$

8. If the ground state wavefunction of a particle moving in a one dimensional potential is proportional to $\exp(-x^2/2)\cosh(\sqrt{2}x)$, then the potential in suitable units such that $\hbar = 1$, is proportional to

(a) x^2 ✓(b) $x^2 - 2\sqrt{2}x\tanh(\sqrt{2}x)$ **[JEST 2017]**

(c) $x^2 - 2\sqrt{2}x\tan(\sqrt{2}x)$ (d) $x^2 - 2\sqrt{2}x\coth(\sqrt{2}x)$

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9. If $\psi(x)$ is n infinitely differentiable function, then $\hat{D}\psi(x)$, where the operator $\hat{D} = \exp\left(ax\frac{d}{dx}\right)$, is

(a) $\psi(x+a)$ (b) $\psi(ae^{a}+x)$ (c) $\psi(e^{a}x)$ (d) $e^{a}\psi(x)$ **[JEST 2018]**

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10. Consider two canonically conjugate operators $\hat{X}$ and $\hat{Y}$ such that $\left[\hat{X},\hat{Y}\right] = i\hbar I$, where I is identify operator. If $\hat{X} = \alpha_{11}\hat{Q}_1 + \alpha_{12}\hat{Q}_2$, $\hat{Y} = \alpha_{21}\hat{Q}_1 + \alpha_{22}\hat{Q}_2$, where α_{ij} are complex numbers, and $\left[\hat{Q}_1,\hat{Q}_2\right] = zI$, the value of $\alpha_{11}\alpha_{22} - \alpha_{12}\alpha_{21}$ is **[JEST 2018]**

(a) $i\hbar z$ (b) $i\hbar / z$ (c) $i\hbar$ (d) z

ANSWER KEY

1. (d) **2. (a)** **3. (b)** **4. (b)** **5. (c)** **6. (b)** **7. (a)**

8. (b) **9. (c)** **10. (b)**

TIFR Previous Years' Questions

~~1.~~ 96 Consider a quantum mechanical system with three linear operators $\hat{A}$, $\hat{B}$ and $\hat{C}$, which are related by

$$\hat{A}\hat{B}-\hat{C}=\hat{I}$$

where $\hat{I}$ is the unit operator. If $\hat{A}=\frac{d}{dx}$ and $\hat{B}=x$, then $\hat{C}$ must be **[TIFR 2013]**

(a) zero (b) $\frac{d}{dx}$ (c) $-x\frac{d}{dx}$ (d) $x\frac{d}{dx}$

~~2.~~ 452 Denote the commutator of two matrices A and B by $[A, B]=AB-BA$ and the anti-commutator by $\{A, B\}= AB+BA$. **[TIFR 2017]**

If $\{A, B\}=0$, we can write $[A, BC]=$

(a) $-B[A,C]$ (b) $B\{A,C\}$ (c) $-B\{A,C\}$ (d) $[A,C]B$

ANSWER KEY

1. (d) **2. (c)**

GATE Previous Years' Questions

1. A particle of energy E is incident on a rectangular potential barrier of height V_0 and width b. **[GATE 1999]**

(a) The reflection coefficient for $E<V_0$ tends to unity as b tends to infinity.
(b) The reflection coefficient is zero for some specific values of $E>V_0$.
(c) The transmission coefficient is always zero for $E<V_0$.
(d) The transmission coefficient is zero for some specific values of $E>V_0$.

2. At t = 0, a one dimensional harmonic oscillator of frequency ω is in a state given by

$$\psi(x,0)=\frac{1}{2}u_0(x)+i\sqrt{3}/2\,u_1(x)$$

where u_0 and u_1 are the first two normalized eigenfunctions. **[GATE 1998]**

(a) The probability of finding the oscillator in state u_1 is $\frac{3}{4}$.

(b) The probability of finding the oscillator in the state u_0 is $\frac{1}{2}$.

(c) The average energy of the oscillator is $2\hbar\omega$.

(d) The average energy of the oscillator is $(5\hbar\omega)$.

3. Let a particle move in a potential field given by **[GATE 2000]**

$$V(x)=\begin{cases}\frac{1}{2}m\omega^2x^2 & \text{for } x>0\\ \infty & \text{for } x\le 0\end{cases}$$

The allowed energies of this particle are

(a) $(n+1/2)\hbar\omega$ (b) $(2n+3/2)\hbar\omega$ (c) $(2n+1/2)\hbar\omega$ (d) $(n+5/2)\hbar\omega$

4. A free particle with energy E whose wave-function is a plane wave with wavelength λ enters a regionof constant potential V>0 where the wavelength of the particle is 2λ. The ratio (V/E) is: **[GATE 2003]**

(a) 1/2 (b) 2/3 (c) 3/4 (d) 4/5

5. Which of the following is a valid normalized wave function for a particle in a one dimensional infinite potential well of width L centred at $x=0$? **[GATE 2003]**

(a) $(2/L)[\cos(2\pi x/L)+\sin(2\pi x/L)]$ (b) $(2/L)^{1/2}\sin[n\pi x/L]$ for odd n

(c) $(2/L)^{1/2}\cos[n\pi x/L]$ for odd n (d) $(2/L)\cos(\pi x/L)$

Common data for Q. 6 and Q.7

A particle is confined to the region $0<x<L$ in one dimension. **[GATE 2004]**

6. If the particle is in the first excited state, then the probability of finding the particle is maximum at

(a) $x=\frac{L}{6}$ (b) $x=\frac{L}{2}$ (c) $x=\frac{L}{3}$ (d) $x=\frac{L}{4}$ and $\frac{3L}{4}$

7. If the particle is in the lowest energy state, then the probability of finding the particle in the region 0 to $\frac{L}{4}$ is:

(a) $\frac{1}{4}-\frac{1}{(2\pi)}$ (b) $\frac{1}{4}$ (c) $\frac{1}{4}+\frac{1}{(2\pi)}$ (d) $\frac{1}{2}$ **[GATE 2004]**

8. A one-dimensional harmonic oscillator carrying a charge $-q$ is placed in a uniform electric field $\vec{E}$ along positive axis. The corresponding Hamiltonian operator is **[GATE 2005]**

(a) $\frac{\hbar^2}{2m}\frac{d^2}{dx^2}+\frac{1}{2}kx^2+qEx$ (b) $-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}+\frac{1}{2}kx^2-qEx$

(c) $-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}+\frac{1}{2}kx^2+qEx$ (d) $-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}+\frac{1}{2}kx^2-qEx$

9. A free particle is moving in +x direction with a linear momentum p. The wavefunction of the particle normalised in a length L is: **[GATE 2006]**

(a) $\frac{1}{\sqrt{L}}\sin\frac{p}{h}x$ (b) $\frac{1}{\sqrt{L}}\cos\frac{p}{h}x$ (c) $\frac{1}{\sqrt{L}}e^{-i\frac{p}{h}x}$ (d) $\frac{1}{\sqrt{L}}e^{i\frac{p}{h}x}$

Common data for Q. 10, Q. 11 and Q. 12

A particle of mass m is confined in the ground state of a one-diemensional box, extending from $x=-2L$ to $x=+2L$. The wavefunction of the particle in this state is $\psi(x)=\psi_0\cos\frac{\pi x}{4L}$. Where ψ_0 is a constant

10. The normalization factor ψ_0 of this wavelength is: **[GATE 2007]**

(a) $\sqrt{\frac{2}{L}}$ (b) $\sqrt{\frac{1}{4L}}$ (c) $\sqrt{\frac{1}{2L}}$ (d) $\sqrt{\frac{1}{L}}$

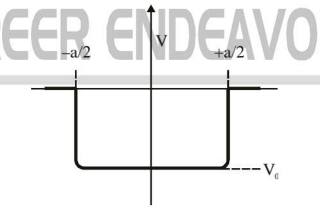
11. The energy eigenvalue corresponding to this state is:

(a) $\frac{\hbar^2\pi^2}{2mL^2}$ (b) $\frac{\hbar^2\pi^2}{4mL^2}$ (c) $\frac{\hbar^2\pi^2}{16mL^2}$ (d) $\frac{\hbar^2\pi^2}{32mL^2}$

12. The expectation value of p^2 (p is the momentum operator) in this state is:

(a) 0 (b) $\frac{\hbar^2\pi^2}{32L^2}$ (c) $\frac{\hbar^2\pi^2}{16L^2}$ (d) $\frac{\hbar^2\pi^2}{8L^2}$

13. There are only three bound states for a particle of mass m in a one-dimensional potential well of the form shown in the figure. The depth V_0 of the potential satisfies **[GATE 2007]**

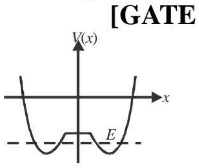


(a) $\frac{2\pi^2\hbar^2}{ma^2}<V_0<\frac{9\pi^2\hbar^2}{2ma^2}$ (b) $\frac{\pi^2\hbar^2}{ma^2}<V_0<\frac{2\pi^2\hbar^2}{ma^2}$

(c) $\frac{2\pi^2\hbar^2}{ma^2}<V_0<\frac{8\pi^2\hbar^2}{ma^2}$ (d) $\frac{2\pi^2\hbar^2}{ma^2}<V_0<\frac{50\pi^2\hbar^2}{ma^2}$

14. A particle with energy E is in a time-independent double well potential as shown in the figure. Which of the following statements about the particle is NOT correct? **[GATE 2007]**

(a) The particle will always be in a bound state.
(b) The probability of finding the particle in one well will be time-dependent.
(c) The particle will be confined to any one of the wells.
(d) The particle can tunnel from one well to the other, and back.



15. The energy levels of a particle of mass m in a potential of the form

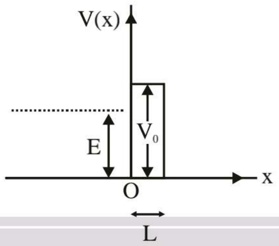
$$V(x) = \infty, \qquad x \le 0$$
$$= \frac{1}{2} m\omega^2 x^2, \qquad x > 0$$

are given, in terms of quantum number $n = 0, 1, 2, 3..........$, by **[GATE 2007]**

(a) $\left(n + \frac{1}{2}\right)\hbar\omega$ (b) $\left(2n + \frac{1}{2}\right)\hbar\omega$ (c) $\left(2n + \frac{3}{2}\right)\hbar\omega$ (d) $\left(n + \frac{3}{2}\right)\hbar\omega$

Common data for Q. 16, Q. 17 and Q. 18

A beam of identical particles of mass m and energy E is incident from left on a potential barrier of width L (between $0 < x < L$) and height V_0 as shown in the figure $(E < V_0)$ **[GATE 2008]**



For $x > L$, there is tunneling with a transmission coefficient $T > 0$. Let A_0, A_R and A_T denote the amplitudes for the incident, reflected and the transmitted waves, respectively.

16. Throughout 0<x<L, the wave -function
(a) Can be chosen to be real (b) Is exponentially decaying.
(c) Is generally complex (d) Is zero.

17. Let the probability current associated with the incident wave be S_0. Let R be the reflection coefficient. Then
(a) The probabiltiy current vanishes in the classically forbidden region.
(b) The probability current is TS_0 for $x > L$.
(c) For, x<0, the probability current is S_0(I+R)
(d) For x>L, the probability current is complex.

18. The ratio of the reflected to the incident amplitude $\frac{A_R}{A_0}$ is:

(a) $1 - \frac{A_R}{A_0}$ (b) $\sqrt{1-T}$ in magnitude

(c) A real negative number (d) $\sqrt{\left(1 - \left|\frac{A_T}{A_0}\right|^2\right)\frac{E}{V_0 - E}}$

19. A particle is placed in a one dimensional box of size L along the x-axis (0<x<L). Which of the following is true? **[GATE 2008]**

(a) In the ground state, the probability of finding the particle in the interval $\left(\frac{L}{4}, \frac{3L}{4}\right)$ is half.

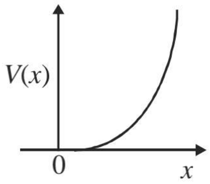
(b) In the first excited state, the probability of finding the particle in the interval $\left(\frac{L}{4}, \frac{3L}{4}\right)$ is half. This also holds for states with $n = 4, 6, 8, ...$

(c) For an arbitory state $|\psi\rangle$, the probability of finding the particle in the left half of the well is half.

(d) In the ground state, the particle has a definite momentum.

20. A particle of mass m is confined in the potential,

$$V(x)=\begin{cases}\frac{1}{2}m\omega^2x^2 & \text{for } x>0,\\ \infty & \text{for } x\le 0\end{cases}$$



Let the wavefunction of the particle be given by

$$\psi(x)=-\frac{1}{\sqrt{5}}\psi_0+\frac{2}{\sqrt{5}}\psi_1,$$

where ψ_0 and ψ_1 are the eigenfunctions of the ground state and the first excited state respectively. The expectation value of the energy is: **[GATE 2010]**

(a) $\frac{31}{10}\hbar\omega$ (b) $\frac{25}{10}\hbar\omega$ (c) $\frac{13}{10}\hbar\omega$ (d) $\frac{11}{10}\hbar\omega$

Common data for Q.21 and Q. 22

In a one-dimensional harmonic oscillator, φ_0,φ_1 and φ_2 are respectively the ground, first and the second excited states. These three states are normalized and are orthogonal to one another. ψ_1 and ψ_2 are two states defined by $\psi_1=\varphi_0-2\varphi_1+3\varphi_2;\ \psi_2=\varphi_0-\varphi_1+\alpha\varphi_2$. Where α is a constant. **[GATE 2011]**

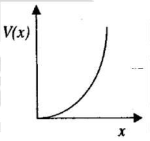
21. The value of α for which ψ_2 is orthogonal to ψ_1 is:

(a) 2 (b) 1 (c) –1 (d) –2

22. For the value of α determined in Q.21, the expectation value of energy of oscillator in the state ψ_2 is:

(a) $\hbar\omega$ (b) $3\hbar\omega/2$ (c) $3\hbar\omega$ (d) $9\hbar\omega/2$

23. A particle is constrained to move in a truncated harmonic potential well $(x>0)$ as shown in the figure. Which one of the following statements is CORRECT ? **[GATE 2012]**



(a) The parity of the first excited state is even (b) The parity of the ground state is even

(c) The ground state energy is $\frac{1}{2}\hbar\omega$ (d) The first excited state energy is $\frac{7}{2}\hbar\omega$

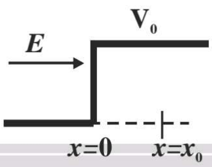
24. A one dimensional harmonic oscillator is in the superposition of number state, $|n\rangle$, given by

$$|\psi\rangle=\frac{1}{2}|2\rangle+\frac{\sqrt{3}}{2}|3\rangle$$

The average energy of the oscillator in the given state is _______ $\hbar\omega$. **[GATE 2014]**

25. Suppose a linear harmonic oscillator of frequency ω and mass m is in the state $|\psi\rangle = \frac{1}{\sqrt{2}}\left[|\psi_0\rangle + e^{i\frac{\pi}{2}}|\psi_1\rangle\right]$ at $t = 0$ where $|\psi_0\rangle$ and $|\psi_1\rangle$ are the ground and the first excited states, respectively. The value of $\langle\psi|x|\psi\rangle$ in the units of $\sqrt{\frac{\hbar}{m\omega}}$ at $t = 0$ is_______ **[GATE 2015]**

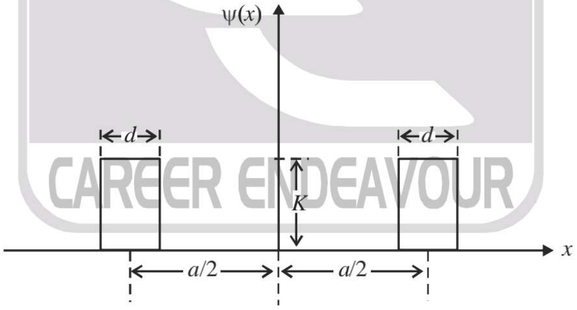
26. A particle of mass m and energy E, moving in the positive x direction, is incident on a step potential at $x = 0$, as indicated in the figure. The height of the potential is V_0, where $V_0 > E$. At $x = x_0$, where $x_0 > 0$, the probability of the finding the electron is $\frac{1}{e}$ times the probability of finding it at $x = 0$. If $\alpha = \sqrt{\frac{2m(V_0 - E)}{\hbar^2}}$, the value of x_0 is **[GATE 2016]**



(a) $\frac{2}{\alpha}$ (b) $\frac{1}{\alpha}$ (c) $\frac{1}{2\alpha}$ (d) $\frac{1}{4\alpha}$

27. A free electron of energy 1 eV is incident upon a one-dimensional finite potential step of height 0.75 eV. The probability of its reflection from the barrier is _______ (up to two decimal places). **[GATE 2017]**

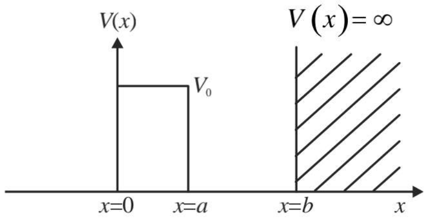
28. The wave function $\psi(x)$ of a particle is as shown below **[GATE 2019]**



Here, K is a constant, and $a > d$. The position uncertainty (Δx) of the particle is

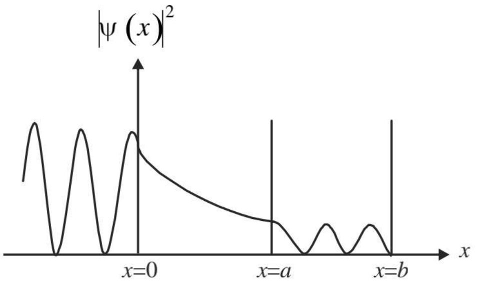
(a) $\sqrt{\frac{a^2 + 3d^2}{12}}$ (b) $\sqrt{\frac{3a^2 + d^2}{12}}$ (c) $\sqrt{\frac{d^2}{6}}$ (d) $\sqrt{\frac{d^2}{24}}$

29. Consider a potential barrier $V(x)$ of the form: **[GATE 2019]**

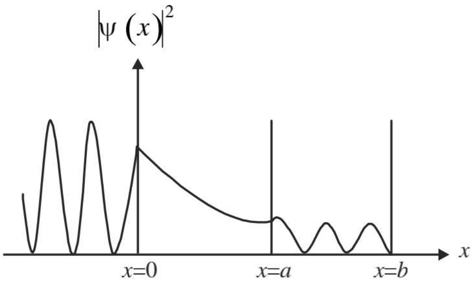


where V_0 is a constant. For particles of energy $E < V_0$ incident on this barrier from the left which of the following schematic diagrams best represents the probability density $|\psi(x)|^2$ as a function of *x*.

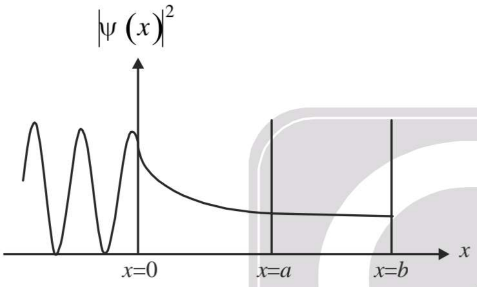
(a)



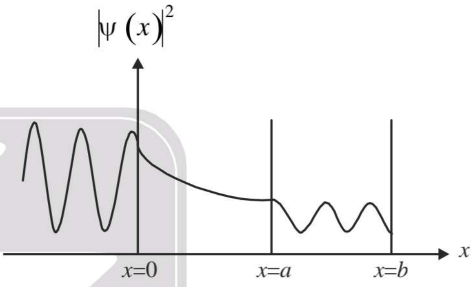
(b)



(c)



(d)



30. Consider the motion of a particle along the *x*-axis in a potential $V(x) = F|x|$. Its ground state energy E_0 is estimated using the uncertainty principle. Then E_0 is proportional to **[GATE 2019]**

(a) $F^{1/3}$ (b) $F^{1/2}$ (c) $F^{2/5}$ (d) $F^{2/3}$

ANSWER KEY

1. (a, b) **2. (a)** **3. (b)** **4. (c)** **5. (c)** **6. (d)** **7. (a)**
8. (b) **9. (d)** **10. (c)** **11. (d)** **12. (c)** **13. (a)** **14. (b)**
15. (c) **16. (b)** **17. (b)** **18. (b)** **19. (c)** **20. (a)** **21. (c)**
22. (b) **23. (d)** **24. (3.25)** **25. (0)** **26. (c)** **27. (0.10 to 0.12)**
28. (b) **29. (a)** **30. (d)**

CSIR-UGC-NET Previous Years' Questions

1. Consider a particle in a one dimensional potential that satisfies $V(x)=V(-x)$. Let $|\psi_0\rangle$ and $|\psi_1\rangle$ denote the ground and the first excited states, respectively, and let $|\psi\rangle=\alpha_0|\psi_0\rangle+\alpha_1|\psi_1\rangle$ be a normalized state with α_0 and α_1 being real constants. The expectation value $\langle x\rangle$ of the position operator x in the state $|\psi\rangle$ is given by **[CSIR Dec 2011]**

(a) $\alpha_0^2\langle\psi_0|x|\psi_0\rangle+\alpha_1^2\langle\psi_1|x|\psi_1\rangle$ (b) $\alpha_0\alpha_1\left[\langle\psi_0|x|\psi_1\rangle+\langle\psi_1|x|\psi_0\rangle\right]$

(c) $\alpha_0^2+\alpha_1^2$ (d) $2\alpha_0\alpha_1$

2. Let $|0\rangle$ and $|1\rangle$ denote the normalized eigenstates corresponding to the ground and first excited state of a one dimensional harmonic oscillator. The uncertainty Δp in the state $\frac{1}{\sqrt{2}}(|0\rangle+|1\rangle)$, is: **[CSIR Dec 2011]**

(a) $\Delta p=\sqrt{\hbar m\omega}/2$ (b) $\Delta p=\sqrt{\hbar m\omega/2}$ (c) $\Delta p=\sqrt{\hbar m\omega}$ (d) $\Delta p=\sqrt{2\hbar m\omega}$

3. A particle in one-dimension is in the potental

$$V(x)=\begin{cases}\infty & \text{if } x<0\\ -V_0 & \text{if } 0\le x\le \ell\\ 0 & \text{if } x>\ell\end{cases}$$

If there is at least one bound state, the minimum depth of the potential is: **[CSIR June 2012]**

(a) $\frac{\hbar^2\pi^2}{8m\ell^2}$ (b) $\frac{\hbar^2\pi^2}{2m\ell^2}$ (c) $\frac{2\hbar^2\pi^2}{m\ell^2}$ (d) $\frac{\hbar^2\pi^2}{m\ell^2}$

4. Let $|0\rangle$ and $|1\rangle$ denote the normalized eigenstates corresponding to the ground and the first excited states of a one-dimensional harmonic oscillator. The uncertainty Δx in the state $\frac{1}{\sqrt{2}}(|0\rangle+|1\rangle)$ is: **[CSIR Dec 2012]**

(a) $\Delta x=\sqrt{\hbar/2m\omega}$ (b) $\Delta x=\sqrt{\hbar/m\omega}$ (c) $\Delta x=\sqrt{2\hbar/m\omega}$ (d) $\Delta x=\sqrt{\hbar/4m\omega}$

5. Consider the normalized state $|\psi\rangle$ of a particle in a one-dimensional harmonic oscillator: $|\psi\rangle=b_1|0\rangle+b_2|1\rangle$ where $|0\rangle$ and $|1\rangle$ denote the ground and first excited states respectively, and b_1 and b_2 are real constants. The expectation value of the displacement x in the state $|\psi\rangle$ will be a minimum when **[CSIR June 2013]**

(a) $b_2=0,\ (b_1=1)$ (b) $b_2=\frac{1}{\sqrt{2}}b_1$ (c) $b_2=\frac{1}{2}b_1$ (d) $b_2=-b_1$

6. A particle is in the ground state of an infinite square well potential given by,

$$V(x)=\begin{cases}0 & \text{for } -a\le x\le a\\ \infty & \text{otherwise}\end{cases}$$

The probability to find the particle in the interval between $-\frac{a}{2}$ and $\frac{a}{2}$ is **[CSIR Dec 2013]**

(a) $\frac{1}{2}$ (b) $\frac{1}{2}+\frac{1}{\pi}$ (c) $\frac{1}{2}-\frac{1}{\pi}$ (d) $\frac{1}{\pi}$

7. A particle in the infinite square well

$$V(x)=\begin{cases}0 & 0<x<a\\ \infty & \text{otherwise}\end{cases}$$

is prepared in a state with the wavefunction

$$\psi(x)=\begin{cases}A\sin^3\left(\dfrac{\pi x}{a}\right) & 0<x<a\\ 0 & \text{otherwise}\end{cases}$$

The expectation value of the energy of the particle is **[CSIR June 2014]**

(a) $\dfrac{5\hbar^2\pi^2}{2ma^2}$ (b) $\dfrac{9\hbar^2\pi^2}{2ma^2}$ (c) $\dfrac{9\hbar^2\pi^2}{10ma^2}$ (d) $\dfrac{\hbar^2\pi^2}{2ma^2}$

8. Let $|\psi\rangle = c_0|0\rangle + c_1|1\rangle$ (where c_0 and c_1 are constants with $c_0^2+c_1^2=1$) be a linear combination of the wavefunctions of the ground and first excited states of the one-dimensional harmonic oscillator. For what value of c_0 is the expectation value $\langle x\rangle$ a maximum? **[CSIR Dec 2014]**

(a) $\langle x\rangle=\sqrt{\dfrac{\hbar}{m\omega}},\ c_0=\dfrac{1}{\sqrt{2}}$ (b) $\langle x\rangle=\sqrt{\dfrac{\hbar}{2m\omega}},\ c_0=\dfrac{1}{2}$

(c) $\langle x\rangle=\sqrt{\dfrac{\hbar}{2m\omega}},\ c_0=\dfrac{1}{\sqrt{2}}$ (d) $\langle x\rangle=\sqrt{\dfrac{\hbar}{m\omega}},\ c_0=\dfrac{1}{2}$

9. The ground state energy of a particle of mass m in the potential $V(x)=V_0\cosh\left(\dfrac{x}{L}\right)$, where L and V_0 are constants (with $V_0 \gg \dfrac{\hbar^2}{2mL^2}$) is approximately **[CSIR Dec. 2015]**

(a) $V_0+\dfrac{\hbar}{L}\sqrt{\dfrac{2V_0}{m}}$ (b) $V_0+\dfrac{\hbar}{L}\sqrt{\dfrac{V_0}{m}}$

(c) $V_0+\dfrac{\hbar}{4L}\sqrt{\dfrac{V_0}{m}}$ (d) $V_0+\dfrac{\hbar}{2L}\sqrt{\dfrac{V_0}{m}}$

10. The state of a particle of mass m in a one-dimensional rigid box in the interval 0 to L is given by the normalised wavefunction **[CSIR June 2016]**

$$\psi(x)=\sqrt{\frac{2}{L}}\left(\frac{3}{5}\sin\left(\frac{2\pi x}{L}\right)+\frac{4}{5}\sin\left(\frac{4\pi x}{L}\right)\right).$$

If its energy is measured, the possible outcomes and the average value of energy are, respectively.

(a) $\dfrac{h^2}{2mL^2},\ \dfrac{2h^2}{mL^2}$ and $\dfrac{73}{50}\dfrac{h^2}{mL^2}$ (b) $\dfrac{h^2}{8mL^2},\ \dfrac{h^2}{2mL^2}$ and $\dfrac{19}{40}\dfrac{h^2}{mL^2}$

(c) $\dfrac{h^2}{2mL^2},\ \dfrac{2h^2}{mL^2}$ and $\dfrac{19}{10}\dfrac{h^2}{mL^2}$ (d) $\dfrac{h^2}{8mL^2},\ \dfrac{2h^2}{mL^2}$ and $\dfrac{73}{200}\dfrac{h^2}{mL^2}$

11. A particle of mass m moves in one-dimension under the influence of the potential $V(x) = -\alpha\delta(x)$, where α is a positive constant. The uncertainty in the product $(\Delta x)(\Delta p)$ in its ground state is

(a) $2\hbar$ (b) $\frac{\hbar}{2}$ (c) $\frac{\hbar}{\sqrt{2}}$ (d) $\sqrt{2}\hbar$ **[CSIR June 2016]**

12. Let $a = \frac{1}{\sqrt{2}}(x+ip)$ and $a^\dagger = \frac{1}{\sqrt{2}}(x-ip)$ be the lowering and raising operators of a simple harmonic oscillator in units where the mass, angular frequency and $\hbar$ have been set to unity. If $|0\rangle$ is the ground state of the oscillator and λ is a complex constant, the expectation value of $\langle\psi|x|\psi\rangle$ in the state $|\psi\rangle = \exp(\lambda a^\dagger - \lambda^* a)|0\rangle$, is **[CSIR Dec. 2016]**

(a) $|\lambda|$ (b) $\sqrt{|\lambda|^2 + \frac{1}{|\lambda|^2}}$ (c) $\frac{1}{\sqrt{2}i}(\lambda - \lambda^*)$ (d) $\frac{1}{\sqrt{2}}(\lambda + \lambda^*)$

13. A particle in one dimension is in a potential $V(x) = A\delta(x-a)$. Its wavefunction $\psi(x)$ is continuous everywhere. The discontinuity in $d\psi/dx$ at $x = a$ is **[CSIR Dec. 2016]**

(a) $\frac{2m}{\hbar^2}A\psi(a)$ (b) $A(\psi(a)-\psi(-a))$ (c) $\frac{\hbar^2}{2m}A$ (d) 0

14. If the root-mean-squared momentum of a particle in the ground state of a one-dimensional simple harmonic potential is p_0, then its root-mean-squared momentum in the first excited state is **[CSIR June 2017]**

(a) $p_0\sqrt{2}$ (b) $p_0\sqrt{3}$ (c) $p_0\sqrt{2/3}$ (d) $p_0\sqrt{3/2}$

15. Consider a potential barrier *A* of height V_0 and width b, and another potential barrier B of height $2V_0$ and the same width b. The ratio T_A/T_B of tunnelling probabilities T_A and T_B, through barriers *A* and *B* respectively, for a particle of energy $V_0/100$, is best approximated by **[CSIR June 2017]**

(a) $\exp\left[\left(\sqrt{1.99}-\sqrt{0.99}\right)\sqrt{\frac{8mV_0b^2}{\hbar^2}}\right]$ (b) $\exp\left[\left(\sqrt{1.98}-\sqrt{0.98}\right)\sqrt{\frac{8mV_0b^2}{\hbar^2}}\right]$

(c) $\exp\left[\left(\sqrt{2.99}-\sqrt{0.99}\right)\sqrt{\frac{8mV_0b^2}{\hbar^2}}\right]$ (d) $\exp\left[\left(\sqrt{2.98}-\sqrt{0.98}\right)\sqrt{\frac{8mV_0b^2}{\hbar^2}}\right]$

16. The state vector of a one-dimensional simple harmonic oscillator of angular frequency ω, at time $t = 0$, is given by $|\psi(0)\rangle = \frac{1}{\sqrt{2}}\left[|0\rangle + |2\rangle\right]$, where $|0\rangle$ and $|2\rangle$ are the normalized ground state and the second excited state, respectively. The minimum time t after which the state vector $|\psi(t)\rangle$ is orthogonal to $|\psi(0)\rangle$, is

(a) $\frac{\pi}{2\omega}$ (b) $\frac{2\pi}{\omega}$ (c) $\frac{\pi}{\omega}$ (d) $\frac{4\pi}{\omega}$ **[CSIR Dec. 2017]**

17. At $t = 0$, the wavefunction of an otherwise free particle confined between two infinite walls at $x = 0$ and $x = L$ is

$$\psi(x, t=0) = \sqrt{\frac{2}{L}}\left(\sin\frac{\pi x}{L} - \sin\frac{3\pi x}{L}\right).$$

Its wavefunction at a later time $t = \frac{mL^2}{4\pi\hbar}$ is **[CSIR June 2018]**

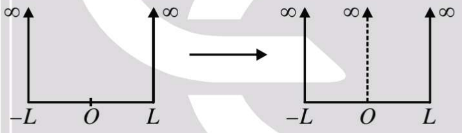
(a) $\sqrt{\frac{2}{L}}\left(\sin\frac{\pi x}{L} - \sin\frac{2\pi x}{L}\right)e^{-i\pi/6}$ (b) $\sqrt{\frac{2}{L}}\left(\sin\frac{\pi x}{L} + \sin\frac{3\pi x}{L}\right)e^{-i\pi/6}$

(c) $\sqrt{\frac{2}{L}}\left(\sin\frac{\pi x}{L} - \sin\frac{3\pi x}{L}\right)e^{-i\pi/8}$ (d) $\sqrt{\frac{2}{L}}\left(\sin\frac{\pi x}{L} + \sin\frac{3\pi x}{L}\right)e^{-i\pi/8}$

18. The product $\Delta x\,\Delta p$ of uncertainties in the position and momentum of a simple harmonic oscillator of mass m and angular frequency ω in the ground state $|0\rangle$, is $\hbar/2$. The value of the product $\Delta x\,\Delta p$ in the state $e^{-i\hat{p}l/\hbar}|0\rangle$, where l is a constant and $\hat{p}$ is the momentum operator) is **[CSIR Dec. 2018]**

(a) $\frac{\hbar}{2}\sqrt{\frac{m\omega l^2}{\hbar}}$ (b) $\hbar$ (c) $\frac{\hbar}{2}$ (d) $\frac{\hbar^2}{m\omega l^2}$

19. A quantum particle of mass m in one dimension, confined to a rigid box as shown in the figure, is in its ground state. An infinitesimally thin wall is very slowly raised to infinity at the centre of the box, in such a way that the system remains in its ground state at all times. Assuming that no energy is lost in raising the wall, the work done on the system when the wall is fully raised, eventually separating the original box into two compartments, is

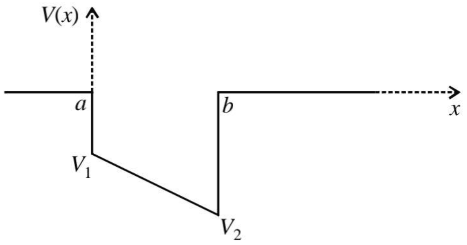


(a) $\frac{3\pi^2\hbar^2}{8mL^2}$ (b) $\frac{\pi^2\hbar^2}{8mL^2}$ (c) $\frac{\pi^2\hbar^2}{2mL^2}$ (d) 0 **[CSIR June 2019]**

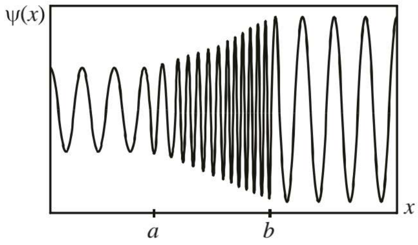
20. The wavefunction of a free particle of mass m, constrained to move in the interval $-L \le x \le L$, is $\psi(x) = A(L + x)(L - x)$, where A is the normalization constant. The probability that the particle will be found to have the energy $\frac{\pi^2\hbar^2}{2mL^2}$ is **[CSIR June 2019]**

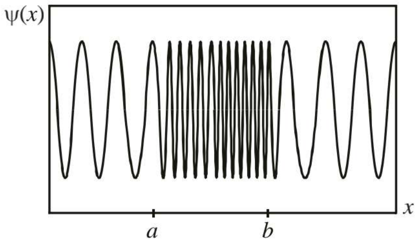
(a) 0 (b) $\frac{1}{\sqrt{2}}$ (c) $\frac{1}{2\sqrt{3}}$ (d) $\frac{1}{\sqrt{\pi}}$

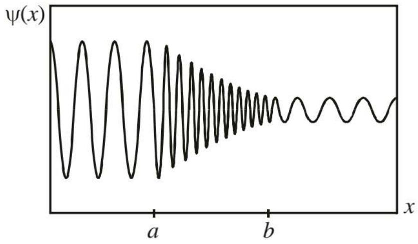
21. A particle of mass m and energy $E > 0$, in one dimension is scattered by the potential shown below.

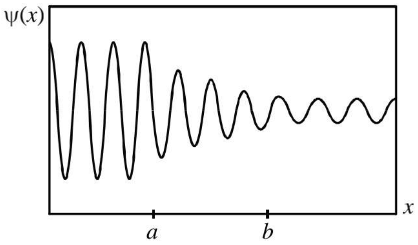


If the particle was moving from $x=-\infty$ to $x=\infty$, which of the following graphs gives the best qualitative representation of the wavefunction of this particle ? **[CSIR June 2019]**

(a) 

(b) 

(c) 

(d) 

22. The energy eigenvalues of a particle of mass m, confined to a rigid one-dimensional box of width L, are $E_n\,(n=1,2,....)$. If the walls of the box are moved very slowly toward each other, the rate of change of time-dependent energy dE_2/dt of the first excited state is **[CSIR Dec. 2019]**

(a) $\frac{E_1}{L}\frac{dL}{dt}$ (b) $\frac{2E_2}{L}\frac{dL}{dt}$ (c) $-\frac{2E_2}{L}\frac{dL}{dt}$ (d) $-\frac{E_1}{L}\frac{dL}{dt}$

23. A particle of mass m is confined to a box of unit length in one dimension. It is described by the wavefunction $\psi(x)=\sqrt{8/5}\,\sin\pi x(1+\cos\pi x)$ for $0\le x\le 1$, and zero outside this interval. The expectation value of energy in this state is **[CSIR Dec. 2019]**

(a) $\frac{4\pi^2}{3m}\hbar^2$ (b) $\frac{4\pi^2}{5m}\hbar^2$ (c) $\frac{2\pi^2}{5m}\hbar^2$ (d) $\frac{8\pi^2}{5m}\hbar^2$

24. The operator $A=\sum_{n=0}^{\infty}|n+1\rangle\langle n|$ is defined in terms of the eigenstates $|n\rangle$ of the number operator of the simple harmonic oscillator. Which of the following relations is obeyed by A and its hermitian conjugate $A^\dagger$? (In the following 1 is the identity operator). **[CSIR Dec. 2019]**

(a) $A^\dagger A=1$ and $AA^\dagger=1$ (b) $A^\dagger A=1$ but $AA^\dagger\ne 1$

(c) $A^\dagger A\ne 1$ and $AA^\dagger=1$ (d) $A^\dagger A\ne 1$ and $AA^\dagger\ne 1$

ANSWER KEY

1. (b) 2. (c) 3. (a) 4. (a) 5. (d) 6. (b) 7. (c)

8. (c) 9. (d) 10. (a) 11. (c) 12. (d) 13. (a) 14. (b)

15. (a) 16. (a) 17. (d) 18. (c) 19. (a) 20. (a) 21. (c)

22. (c) 23. (b) 24. (b)

JEST Previous Years' Questions

1. A quantum mechanical particle in a harmonic oscillator potential has the initial wavefunction $\psi_0(x)+\psi_1(x)$, where ψ_0 and ψ_1 are the real wavefunctions in the ground and first excited states of the harmonic oscillator Hamiltonian. For convenience we take $m=\hbar=\omega=1$ for the oscillator. What is the probability density of finding the particle at x at time t= π? **[JEST 2013]**

(a) $(\psi_1(x)-\psi_0(x))^2$ (b) $(\psi_1(x))^2-(\psi_0(x))^2$

(c) $(\psi_1(x)+\psi_0(x))^2$ (d) $(\psi_1(x))^2+(\psi_0(x))^2$

2. A particle of mass m is contained in a one-dimensional infinite well extending from x = –L/2 to x = L/2. The particle is in its ground state given by $\psi_0(x)=\sqrt{2/L}\cos(\pi x/L)$. The walls of the box are moved suddenly to form a box extending from x = –L to x = L. What is the probability that the particle will be in the ground state after this sudden expansion? **[JEST 2013]**

(a) $(8/3\pi)^2$ (b) 0 (c) $(16/3\pi)^2$ (d) $(4/3\pi)^2$

4. Consider a square well of depth $-V_0$ and width a with $V_0 a$ fixed. Let $V_0\to\infty$ and $a\to 0$. This potential well has **[JEST 2014]**

(a) No bound states (b) 1 bound state (c) 2 bound states (d) Infinitely many bound states

5. Consider a harmonic oscillator in the state $|\psi\rangle = e^{-\frac{|\alpha|^2}{2}}e^{\alpha a^\dagger}|0\rangle$, where $|0\rangle$ is the ground state, $a^\dagger$ is the raising operator and α is a complex number. What is the probability that the harmonic oscillator is in the n-th eigenstate $|n\rangle$? **[JEST 2015]**

(a) $e^{-|\alpha|^2}\dfrac{|\alpha|^{2n}}{n!}$ (b) $e^{-\frac{|\alpha|^2}{2}}\dfrac{|\alpha|^n}{\sqrt{n!}}$ (c) $e^{-|\alpha|^2}\dfrac{|\alpha|^n}{n!}$ (d) $e^{-\frac{|\alpha|^2}{2}}\dfrac{|\alpha|^{2n}}{n!}$

6. A particle of mass m moves in 1-dimensional potential $V(x)$, which vanishes at infinity. The exact ground state eigenfunction is $\psi(x)=A\,\text{sech}(\lambda x)$ where A and λ are constant. The ground state energy eigenvalue of this system is, **[JEST 2015]**

(a) $E=\dfrac{\hbar^2\lambda^2}{m}$ (b) $E=-\dfrac{\hbar^2\lambda^2}{m}$ (c) $E=-\dfrac{\hbar^2\lambda^2}{2m}$ (d) $E=\dfrac{\hbar^2\lambda^2}{2m}$

7. An electron confined within a thin layer of semiconductor may be treated as a free particle inside an infinitely deep one-dimensional potential well. If the difference in energies between the first and the second energy levels is δE, then the thickness of the layer is: **[JEST 2016]**

(a) $\sqrt{\dfrac{3\hbar^2\pi^2}{2m\delta E}}$ (b) $\sqrt{\dfrac{2\hbar^2\pi^2}{3m\delta E}}$ (c) $\sqrt{\dfrac{\hbar^2\pi^2}{2m\delta E}}$ (d) $\sqrt{\dfrac{\hbar^2\pi^2}{m\delta E}}$

8. $\phi_0(x)$ and $\phi_1(x)$ are respectively the orthogonal wavefunctions of the ground and first excited states of a one dimensional simple harmonic oscillator. Consider the normalised wave function $\psi(x)=c_0\,\phi_0(x)+c_1\,\phi_1(x)$, where c_0 and c_1 are real. For what values of c_0 and c_1 will $\langle\psi(x)|x|\psi(x)\rangle$ be maximized ?

(a) $c_0=c_1=+1/\sqrt{2}$ (b) $c_0=-c_1=+1/\sqrt{2}$ **[JEST 2017]**

(c) $c_0=+\sqrt{3}/2,\ c_1=+1/2$ (d) $c_0=+\sqrt{3}/2,\ c_1=-1/2$

9. If $\hat{x}(t)$ be the position operator at a time *t* in the Heisenberg picture for a particle described by the Hamiltonian, $\hat{H}=\dfrac{\hat{p}^2}{2m}+\dfrac{1}{2}m\omega^2\hat{x}^2$, what is $e^{i\omega t}\langle 0|\hat{x}(t)\hat{x}(0)|0\rangle$ in units of $\dfrac{\hbar}{2m\omega}$, where $|0\rangle$ is the ground state ?

[JEST 2017]

10. A one dimensional harmonic oscillator (mass *m* and frequency ω) is in a state ψ such that the only possible outcomes of an energy measurement are E_0, E_1 or E_2, where E_n is the energy is the energy of the *n*-th excited state. If *H* is the Hamiltonian of the oscillator, $\langle\psi|H|\psi\rangle=3\hbar\omega/2$ and $\langle\psi|H^2|\psi\rangle=11\hbar^2\omega^2/4$, then the probability that the energy measurement yields E_0 is **[JEST 2018]**

(a) 1/2 (b) 1/4 (c) 1/8 (d) 0

11. The normalized eigenfunctions and eigenvalues of the Hamiltonian of a particle confined to move between $0\le x\le a$ in one dimension are

$$\psi_n(x)=\frac{2}{a}\sin\frac{n\pi x}{a}$$

and $$E_n=\frac{n^2\pi^2\hbar^2}{2ma^2}$$

respectively. Here 1, 2, 3, ... Suppose the state of the particle is

$$\psi(x)=A\sin\left(\frac{\pi x}{a}\right)\left[1+\cos\left(\frac{\pi x}{a}\right)\right]$$

where *A* is the normalization constant. If the energy of the particle is measured, the probability to get the result as $\dfrac{\pi^2\hbar^2}{2ma^2}$ is $x/100$. What is the value of *x*? **[JEST 2018]**

12. The wave function $\psi(x)=A\exp\left(-\dfrac{b^2x^2}{2}\right)$ (for real constants *A* and *b*) is a normalized eigenfunction of the Schrödinger equation for a particle of mass *m* and energy *E* in a one dimensional potential $V(x)$ such that $V(x)=0$ at $x=0$. Which one of the following is correct ? **[JEST 2019]**

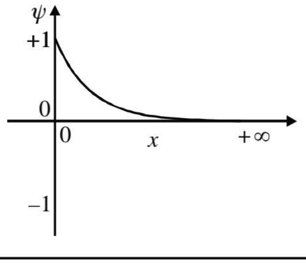
(a) $V=\dfrac{\hbar^2b^4x^2}{m}$ (b) $V=\dfrac{\hbar^2b^4x^2}{2m}$ (c) $E=\dfrac{\hbar^2b^4}{4m}$ (d) $E=\dfrac{\hbar^2b^4}{m}$

13. A quantum particle of mass *m* is in a one dimensional potential of the form

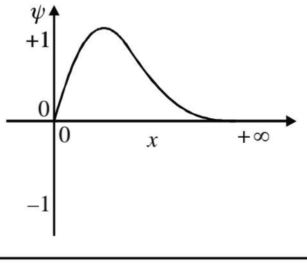
$$V(x)=\begin{cases}\frac{1}{2}m\omega^2x^2, & \text{if}\quad x>0\\ \infty, & \text{if}\quad x\le 0\end{cases},$$

where ω is a constant. Which one of the following represents the possible ground state wave function of the particle ? **[JEST 2019]**

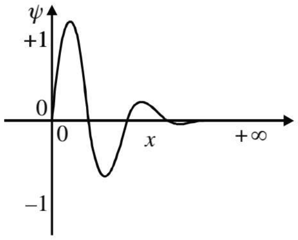
(a)



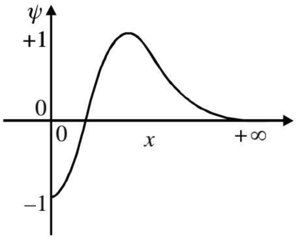
(b)



(c)



(d)



14. A one-dimensional harmonic oscillator is in the state $|\psi\rangle = \sum_{n=0}^{\infty} \frac{1}{\sqrt{n!}} |n\rangle,$ where $|n\rangle$ is the normalized energy eigenstate with eigenvalue $\left(n+\frac{1}{2}\right)\hbar\omega$. Let the expectation value of the Hamiltonian in the state $|\psi\rangle$ be expressed as $\frac{1}{2}\alpha\hbar\omega$. What is the value of α ? **[JEST 2019]**

15. Consider a quantum particle in a one-dimensional box of length L. The coordinates of the leftmost wall of the box is at $x = 0$ and that of the rightmost wall is at $x = L$. The particle is in the ground state at $t = 0$, we suddenly change the length of the box to $3L$ by moving the right wall. What is the probability that the particle is in the ground state of the new system immediately after the change ? **[JEST 2019]**

(a) 0.36 (b) $\frac{9}{8\pi}$ (c) $\frac{81}{64\pi^2}$ (d) $\frac{0.5}{\pi}L$

ANSWER KEY

1. (a) **2. (a)** **3. (c)** **4. (b)** **5. (a)** **6. (c)** **7. (a)**
8. (a) **9. (0001)** **10. (b)** **11. (*)** **12. (b)** **13. (b)** **14. (3)**
15. (c)

TIFR Previous Years' Questions

1. A particle P_1 is confined in a one-dimensional infinite potential well with walls at $x = \pm 1$. Another Particle P_2 is confined in a one-dimensional infinite potential well with walls at $x = 0, 1$. Comparing the two particles, one can conclude that **[TIFR 2010]**

(a) the no. of nodes in the nth excited state of P_1 is twice that of P_2

(b) the no. of nodes in the nth excited state of P_1 is half that of P_2

(c) the energy of the nth level of P_1 is the same as that of P_2

(d) the energy of the nth level of P_1 is one quarter of that of P_2

2. The wave function Ψ of a quantum mechanical system described by a Hamiltonian $\hat{H}$ can be written as a linear combination of Φ_1 and Φ_2 which are the eigenfunctions of $\hat{H}$ with eigenvalues E_1 and E_2 respectively At t = 0, the system is prepared is the state $\Psi_0 = \frac{4}{5}\Phi_1 + \frac{3}{5}\Phi_2$ and then allowed to evolve with time. The wavefunction at time $T = \frac{\frac{1}{2}h}{(E_1 - E_2)}$ will be (accurate to within a phase). **[TIFR 2010]**

(a) $\frac{4}{5}\Phi_1 + \frac{3}{5}\Phi_2$ (b) Φ_1 (c) $\frac{4}{5}\Phi_1 - \frac{3}{5}\Phi_2$ (d) Φ_2

(e) $\frac{3}{5}\Phi_1 + \frac{4}{5}\Phi_2$ (f) $\frac{3}{5}\Phi_1 - \frac{4}{5}\Phi_2$

3. A particle of mass m is placed in the ground state of a one-dimensional harmonic oscillator potential of the form

$$V(x) = \frac{1}{2}kx^2$$

where the stiffness constant k can be varied externally. The ground state wavefunction has the form $\psi(x) \propto \exp(-ax^2\sqrt{k})$ where a is a constant. If, suddenly, the parameter k is changed to 4k, the probability that the particle will remain in the ground state of the new potential is **[TIFR 2011]**

(a) 0.47 (b) 0.06 (c) 0.53 (d) 0.67

(e) 0.33 (f) 0.94

4. A particle in a one-dimensional potential has the wavefunction $\psi(x) = \frac{1}{\sqrt{a}}\exp\left(\frac{-|x|}{a}\right)$ where a is a constant. It follows that for a positive constant V_0, the potential $V(x)$ is **[TIFR 2012]**

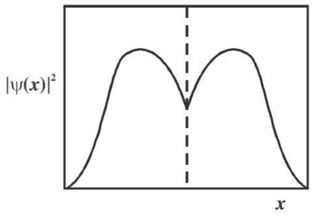
(a) $V_0 x^2$ (b) $V_0|x|$ (c) $-V_0\delta(x)$ (d) $-V_0/|x|$

5. A harmonic oscillator has the wave function, $\psi(x,t) = \frac{1}{5}[3\varphi_0(x,t) - 2\sqrt{2}\varphi_1(x,t) + 2\sqrt{2}\varphi_2(x,t)]$

where $\varphi_n(x,t)$ is the eigenfunction belonging to the n-th energy eigenvalue $\left(n + \frac{1}{2}\right)\hbar\omega$. The expectation value $\langle E\rangle$ of energy for the state $\psi(x,t)$ is **[TIFR 2013]**

(a) $1.58\hbar\omega$ (b) $0.46\hbar\omega$ (c) $\hbar\omega$ (d) $1.46\hbar\omega$

6. A particle is confined to a one-dimensional box of length L. If a vanishingly thin but strongly repulsive partition is introduced in the exact centre of the box, and the particle is allowed to come to its ground state, then the probability density for finding the particle will appear as **[TIFR 2014]**

(a)


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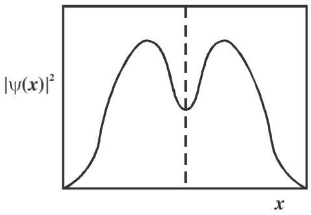
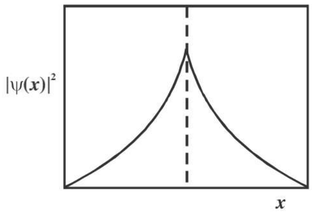


(c)


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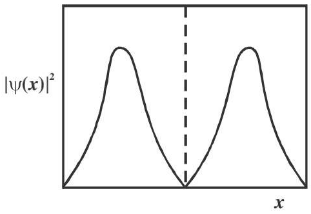


7. A particle is moving in one dimension under a potential $V(x)$ such that, for large positive values of $x, V(x) \approx kx^{\beta}$, where $k > 0$ and $\beta \geq 1$. If the wavefunction in this region has the form $\psi(x) \sim \exp(-x^{\lambda})$, which of the following is true ? **[TIFR 2015]**

(a) $\lambda = \frac{\beta}{2} + 1$ (b) $\lambda = \beta$ (c) $\lambda = 2\beta - 2$ (d) $\lambda = \frac{\beta^2}{2}$

8. A one-dimensional box contains a particle whose ground state energy is $\in$. It is observed that a small disturbance causes the particle to emit a photon of energy $h\nu = 8\in$, after which it is stable. Just before emission, a possible state of the particle in terms of the energy eigenstates $\{\psi_1, \psi_2, ...\}$ would be **[TIFR 2015]**

(a) $\frac{\psi_1 - \psi_2}{\sqrt{2}}$ (b) $\frac{\psi_2 + 2\psi_3}{\sqrt{5}}$ (c) $\frac{-4\psi_4 + 5\psi_5}{\sqrt{41}}$ (d) $\frac{\sqrt{2}\psi_1 - 3\psi_2 + 5\psi_5}{6}$

9. 1000 neutral spinless particles are confined in a one-dimensional box of length 100 nm. At a given instant of time, if 100 of these particle have energy $4\in_0$ and the remaining 900 have energy $225\in_0$, then the number of particles in the left half of the box will be approximately **[TIFR 2015]**

(a) 625 (b) 500 (c) 441 (d) 100

10. A one-dimensional quantum harmonic oscillator is in its ground state **[TIFR 2015]**

$$\psi_0(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} e^{-m\omega x^2/2\hbar}$$

Two experiments, [A] and [B], are performed on the system. In [A], the frequency ω of the oscillator is suddenly doubled, while in [B] the frequency ω is suddenly halved. If p_A and p_B denote the probability in each case that the system is found in its new ground state immediately after the frequency change, which of the following is true ?

(a) $p_A = \sqrt{2}p_B$ (b) $p_A = 2p_B$ (c) $2p_A = p_B$ (d) $p_A = p_B$

11. A one-dimensional harmonic oscillator of mass m and natural frequency ω is in the quantum state

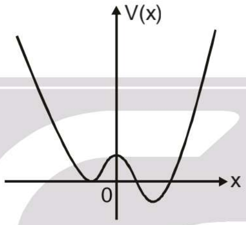
$$|\psi\rangle = \frac{1}{\sqrt{3}}|0\rangle + \frac{i}{\sqrt{3}}|1\rangle + \frac{i}{\sqrt{3}}|2\rangle$$ **[TIFR 2016]**

at time $t = 0$, where $|n\rangle$ denotes the eigenstate with eigenvalue $\left(n+\frac{1}{2}\right)\hbar\omega$. It follows that the exception value $\langle x(t)\rangle$ of the position operator $\hat{x}$ is

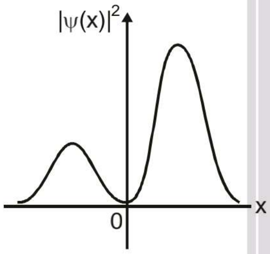
(a) $\langle x(0)\rangle\left[\cos\omega t + \frac{1}{\sqrt{3}}\sin\omega t\right]$ (b) $\langle x(0)\rangle[\cos\omega t - \sin\omega t]$

(c) $\langle x(0)\rangle\left[\cos\omega t - \frac{1}{2}\sin\omega t\right]$ (d) $\langle x(0)\rangle\left[\cos\omega t + \frac{1}{\sqrt{2}}\sin\omega t\right]$

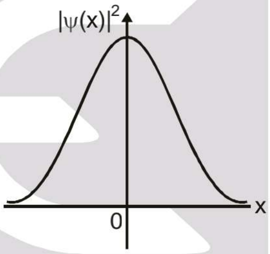
12. A particle is confined inside a one-dimensional potential well $V(x)$, as shown on the right. One of the possible probability distributions $|\psi(x)|^2$ for an energy eigenstate for this particle is **[TIFR 2016]**



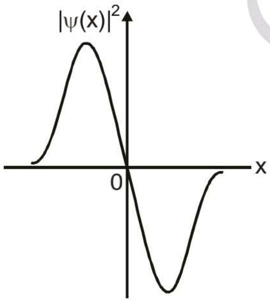
(a)



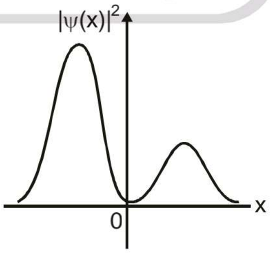
(b)



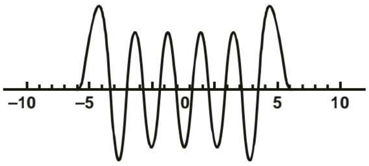
(c)



(d)

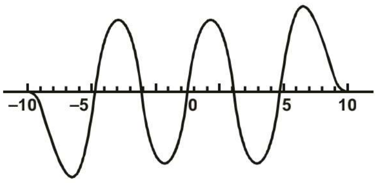


13. Two harmonic oscillators A and B are in excited eigenstates with the same excitation energy E, as measured from their respective ground state energies. The natural frequency of A is twice that of B.

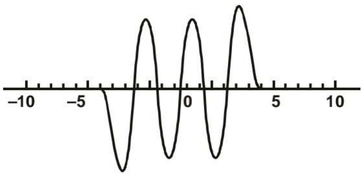


If the wavefunction of B is as sketched in the above picture, which of the following would best represent the wavefunction of A ? **[TIFR 2016]**

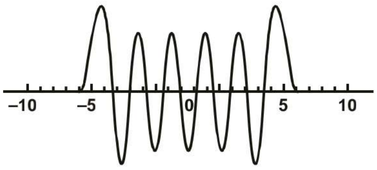
(a)



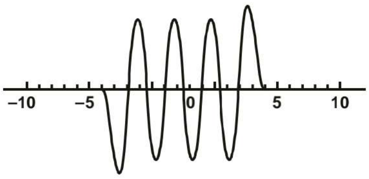
(b)



(c)



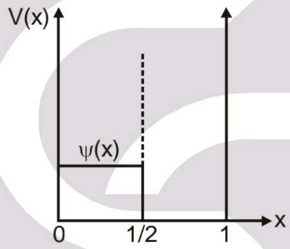
(d)



14. A particle is confined in a one-dimensional box of unit length, i.e. $L = 1$, i.e. in a potential

$$V(x) = \begin{cases} 0 & \text{if} \quad 0 < x < 1 \\ \infty & \quad\text{otherwise} \end{cases}$$

The energy eigenvalues of this particle are denoted $E_0, E_1, E_2, E_3, \ldots$

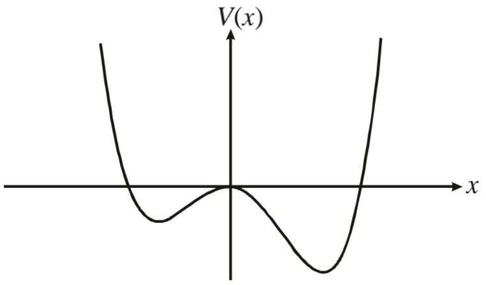


In a particular experiment, the wavefunction of this particle, at $t = 0$, is given by

$$\psi(x) = \begin{cases} \sqrt{2} & \text{if} \quad 0 < x < 1/2 \\ 0 & \quad\text{otherwise} \end{cases}$$

If, simultaneouly, i.e. at $t = 0$, a measurement of the energy of the particle is made, find $100p_3$, where p_3 is the probability that the measurement will yield the energy E_3. **[TIFR 2016]**

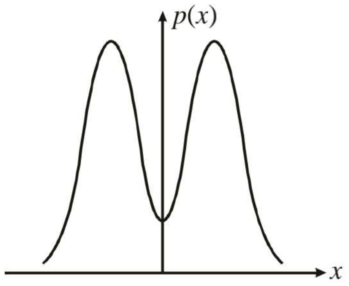
15. Consider the 1-D asymmetric double-well potential $V(x)$ as sketched below.



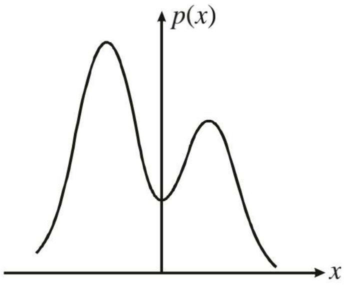
The probability distribution $p(x)$ of a particle in the ground state of this potential is best represented by

[TIFR 2017]

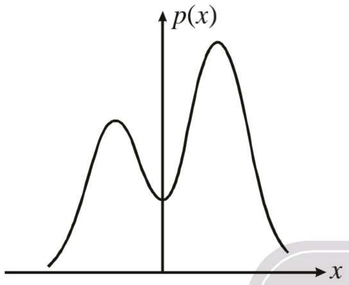
(a)



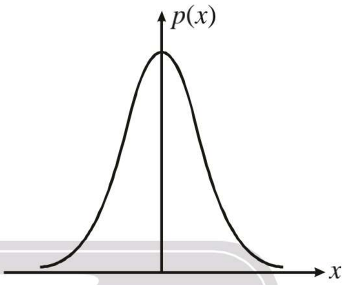
(b)



(c)



(d)



16. A particle of mass m, confined to one dimension x, is in ground state of a harmonic oscillator potential with a normalized wavefunction $\Psi_0(x)=\left(\frac{2a}{\pi}\right)^{1/4}e^{-ax^2}$, where $a=m\omega/2\hbar$. Find the expectation value of x^8 in terms of the parameter a. **[TIFR 2017]**

17. A particle is confined inside a one-dimensional box of length l and left undisturbed for a long time. In the most general case, its wave-function MUST be **[TIFR 2018]**
(a) the ground state of energy
(b) periodic, where l equals an integer number of periods
(c) a linear superposition of the energy eigenfunctions
(d) any one of the energy eigenfunctions

18. The wave-function Ψ of a particle in a one-dimensional harmonic oscillator potential is given by

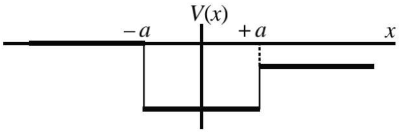
$$\Psi=\left(\frac{1}{\pi l^2}\right)^{1/4}\left(1+\frac{\sqrt{2}x}{l}\right)\exp\left(-\frac{x^2}{2l^2}\right),$$ **[TIFR 2018]**

where $l = 100$ μm. Find the expectation value of the position x of this particle, in μm.

19. A particle in a one-dimensional harmonic oscillator potential is described by a wave-function $\psi(x,t)$. If the wavefunction changes to $\psi(\lambda x,t)$, then the expectation value of kinetic energy T and the potential energy V will change, respectively, to **[TIFR 2018]**

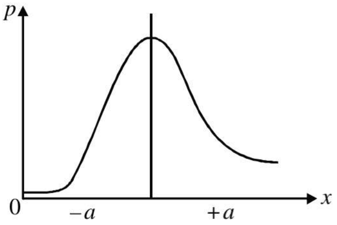
(a) λ^2T and V/λ^2 (b) T/λ^2 and V/λ^2 (c) T/λ^2 and λ^2V (d) λ^2T and λ^2V

20. A particle moving in one dimension, is placed in an asymmetric square well potential $V(x)$ as sketched below.

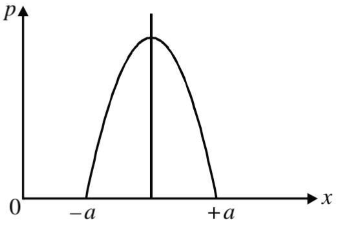


The probability density $p(x)$ in the ground state will most closely resemble **[TIFR 2019]**

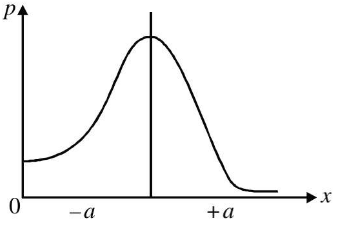
(a)



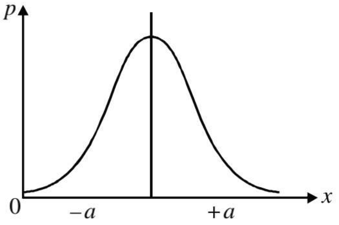
(b)



(c)



(d)



21. An excited gas, consisting of spinless charged particles, is confined in an infinite square well potential of width a, is found to radiate a spectrum whose α line (largest wavelength) has wavelength 816 nm. If the width a of the well is halved to $a/2$, the wavelength of the δ line (fourth-largest wavelength) will be **[TIFR 2019]**

(a) $1.088\ \mu\text{m}$ (b) $0.306\ \mu\text{m}$ (c) $1.224\ \mu\text{m}$ (d) $26.122\ \mu\text{m}$

22. The wave function of a non-relativistic particle of mass m in a one-dimensional potential $V(x)$ has the form

$$\psi(x) = \sqrt{a}\, e^{-a|x|}$$

where $|x|$ denotes the absolute value of the coordinate x. The potential is $V(x) =$ **[TIFR 2019]**

(a) $-\dfrac{\hbar^2}{m}\delta\left(\dfrac{x}{a}\right)$ (b) $\dfrac{\hbar^2 a}{m}\delta(x)$ (c) 0 (d) $-\dfrac{\hbar^2 a}{m}\left\{a + \dfrac{1}{2}\delta(x)\right\}$

23. In a one-dimensional system, the boundary condition that the derivative of the wavefunction $\psi'(x)$ should be continuous at every point is applicable whenever **[TIFR 2019]**

(a) the wavefunction $\psi(x)$ is itself continuous everywhere.
(b) there is a bound state and the potential is piecewise continuous.
(c) there are bound or scattering states with definite momentum.
(d) there is a bound state and the potential has no singularity anywhere.

24. At time $t = 0$, the wavefunction of a particle in a harmonic oscillator potential of natural frequency ω is given by

$$\psi(0) = \frac{1}{5}\left[3\varphi_0 - 2\sqrt{2}\,\varphi_1 + 2\sqrt{2}\,\varphi_2\right],$$

where $\varphi_n(x)$ denotes the eigenfunction belonging to the n^{th} eigenvalue of energy. At time $t = \tau$, the wavefunction is found to be

$$\psi(\tau) = -\frac{i}{5}\left[3\varphi_0 + 2\sqrt{2}\,\varphi_1 + 2\sqrt{2}\,\varphi_2\right]$$

The minimum value of τ is **[TIFR 2019]**

(a) $2\pi/3\omega$ (b) $\pi/2\omega$ (c) π/ω (d) $2\pi/\omega$

ANSWER KEY

1. (d) **2.** (c) **3.** (f) **4.** (c) **5.** (d) **6.** (a) **7.** (a)

8. (b) **9.** (b) **10.** (d) **11.** (d) **12.** (a) **13.** (b) **14.** (0)

15. (c) **16.** $\left[105/\left(256\times a^{4}\right)\right]$ **17.** (c) **18.** (071) **19.** (a) **20.** (a)

21. (c) **22.** (a) **23.** (d) **24.** (c)

GATE Previous Years' Questions

1. A system is known to be in a state described by the wave function **[GATE 1998]**

$$\psi(\theta,\phi)=\frac{1}{\sqrt{30}}\left(5Y_4^0+Y_6^0-2Y_6^3\right)$$

where $Y_\ell^m(\theta,\phi)$ are the spherical harmonics. The probability of finding the system in a state with $m=0$ is

(a) zero (b) 2/15 (c) 1/4 (d) 13/15

2. In a Stern-Gerlach experiment the atomic beam whose angular momentum state is to be determined, must travel through **[GATE 2000]**

(a) homogeneous radio frequency magnetic field
(b) homogeneous static magnetic field
(c) inhomogeneous static magnetic field
(d) inhomogeneous radio frequency magnetic field

3. A spin ½ particle with g>0, is subjected to a magnetic field $\vec{H}=H_0\hat{z}$. If the quantization axis is along $+\hat{z}$, then the minimum energy eigenstate is given by, **[GATE 2000]**

(a) $|\downarrow\rangle$ (b) $\frac{1}{\sqrt{2}}|\uparrow\rangle-|\downarrow\rangle$ (c) $|\uparrow\rangle$ (d) $\frac{1}{\sqrt{2}}|\uparrow\rangle+|\downarrow\rangle$

4. If a spin 1 particle is in the state $|m=0\rangle$ with respect to a quantization axis $\hat{n}$, which of the following is correct? **[GATE 2000]**

(a) $\langle\vec{S}\rangle=0$ (b) $\langle\vec{S}\rangle=\hat{n}$ (c) $\langle\vec{S}\rangle=\sqrt{2}\hat{n}$ (d) $\langle\vec{S}\rangle=-\hat{n}$

5. The Hamiltonian of a particle is given by $H=\frac{p^2}{2m}+V(|\vec{r}|)+\phi(|\vec{r}|)\vec{L}\cdot\vec{S}$, where $\vec{S}$ is the spin, $V(|\vec{r}|)$ and $\phi(|\vec{r}|)$ are potential functions and $\vec{L}=\vec{r}\times\vec{p}$ is the angular momentum. The Hamiltonian does NOT commute with **[GATE 2004]**

(a) $\vec{L}+\vec{S}$ (b) $\vec{S}^2$ (c) L_z (d) $\vec{L}^2$

6. Let $\vec{L}=(L_x,L_y,L_z)$ denote the orbital angular momentum operators of a particle and let $L_+=L_x+iL_y$ and $L_-=L_x-iL_y$. The particle is in an eigenstate of L^2 and L_z eigenvalues $\ell(\ell+1)\hbar^2$ and $\hbar\ell$ respectively. The expectation value of L_+L_- in this state is: **[GATE 2001]**

(a) $\ell\hbar^2$ (b) $2\ell\hbar^2$ (c) 0 (d) $\ell\hbar$

7. An electron is in a state spin wave function $\phi_s=\begin{pmatrix}\sqrt{3}/2\\1/2\end{pmatrix}$ in the S_z representation. What is the probability of finding the z-componenet of its spin along the $-\hat{z}$ direction? **[GATE 2002]**

(a) 0.75 (b) 0.50 (c) 0.35 (d) 0.25

8. In a Stern-Gerlach experiment, the magnetic field is in +z direction. A particle comes out of this experiment in $|+\hat{Z}\uparrow\rangle$ state. Which of the following statements is true? **[GATE 2002]**

(a) The particle has a definite value of the y-component of the spin angular momentum.
(b) The particle has a definite value of the square of the spin angular momentum.
(c) The particle has a definite value of x-components of spin angular momentum.
(d) The particle has definite values of x- and y-components of spin angular momentum.

9. A spin half particle is in the state $S_z = \hbar/2$. The expectation values of S_x, S_x^2, S_y, S_y^2 are given by **[GATE 2003]**

(a) $0, 0, \hbar^2/4, \hbar^2/4$ (b) $0, \hbar^2/4, \hbar^2/4, 0$ (c) $0, \hbar^2/4, 0, \hbar^2/4$ (d) $\hbar^2/4, \hbar^2/4, 0, 0$

10. The commutator, $\left[L_z, Y_{\ell m}(\theta, \phi)\right]$ where L_z is the *z* component of the orbital angular momentum and $Y_{\ell m}(\theta, \phi)$ is a spherical harmonic, is: **[GATE 2005]**

(a) $\ell(\ell+1)\hbar$ (b) $-m\hbar$ (c) $m\hbar$ (d) $+\ell\hbar$

11. For a spin 1/2 particle, the expectation value of $S_x S_y S_z$. where S_x, S_y and S_z are spin operators, is **[GATE 2005]**

(a) $\frac{i\hbar^3}{8}$ (b) $-\frac{i\hbar^3}{8}$ (c) $\frac{i\hbar^3}{16}$ (d) $-\frac{i\hbar^3}{16}$

12. Three operators X, Y and Z satisfy the commutation relations

$$[X, Y] = i\hbar Z, [Y, Z] = i\hbar X \text{ and } [Z, X] = i\hbar Y$$

The set of all possible eigenvalues of the operator Z, in units of $\hbar$, is **[GATE 2007]**

(a) $\{0, \pm 1, \pm 2, \pm 3, ...\}$ (b) $\left\{\frac{1}{2}, 1, \frac{3}{2}, 2, \frac{5}{2}, ...\right\}$

(c) $\left\{0, \pm\frac{1}{2}, \pm 1, \pm\frac{3}{2}, \pm 2, \pm\frac{3}{2}, ...\right\}$ (d) $\left\{-\frac{1}{2}, +\frac{1}{2}\right\}$

13. Let $|\psi_0\rangle$ denote the ground state of the hydrogen atom. Choose the correct statement from those given below: **[GATE 2008]**

(a) $\left[L_x, L_y\right]|\psi_0\rangle = 0$ (b) $J^2|\psi_0\rangle = 0$ (c) $\vec{L}\cdot\vec{S}|\psi_0\rangle \neq 0$ (d) $\left[S_x, S_y\right]|\psi_0\rangle = 0$

14. The spin function of a free particle, in the basis in which S_Z is diagonal, can be written as $\begin{pmatrix}1\\0\end{pmatrix}$ and $\begin{pmatrix}0\\1\end{pmatrix}$ with eigenvalues $+\frac{\hbar}{2}$ and $\frac{\hbar}{2}$, respectively. In the given basis, the normalized eigenfunction of S_y with eigenvalue $-\frac{\hbar}{2}$

(a) $\frac{1}{\sqrt{2}}\begin{pmatrix}1\\-i\end{pmatrix}$ (b) $\frac{1}{\sqrt{2}}\begin{pmatrix}0\\i\end{pmatrix}$ (c) $\frac{1}{\sqrt{2}}\begin{pmatrix}i\\0\end{pmatrix}$ (d) $\frac{1}{\sqrt{2}}\begin{pmatrix}i\\1\end{pmatrix}$ **[GATE 2009]**

15. For a spin *s* particle, in the eigen basis of S^2, S_z the expectation value $\langle sm \mid S_x^2 \mid sm\rangle$ is

[GATE 2010]

(a) $\frac{\left[s(s+1) - m^2\right]\hbar^2}{2}$ (b) $\left[s(s+1) - 2m^2\right]\hbar^2$

(c) $\left[s(s+1) - m^2\right]\hbar^2$ (d) $m^2\hbar^2$

16. If L_x, L_y and L_z are the x, y and z components of angular momentum operator *L*, the commutator $\left[L_x L_y, L_z\right]$ is equal to **[GATE 2011]**

(a) $i\hbar\left(L_x^2 + L_y^2\right)$ (b) $i\hbar\left(L_x^2 - L_y^2\right)$ (c) $2i\hbar L_z$ (d) zero

17. Which one of the following commutation relations is not correct? **[GATE 2013]**

(a) $\left[L^2, L_z\right] = 0$ (b) $\left[L_x, L_y\right] = i\hbar L_z$ (c) $\left[L_z, L_+\right] = \hbar L_+$ (d) $\left[L_z, L_-\right] = \hbar L_-$

18. A spin-half particle is in a linear superposition $0.8|\uparrow\rangle + 0.6|\downarrow\rangle$ of its spin-up and spin-down states. If $|\uparrow\rangle$ and $|\downarrow\rangle$ are the eigen states of σ_z then what is the expectation value, up to one decimal place, of the operator $10\sigma_z + 5\sigma_x$? .... **[GATE 2013]**

19. If $\vec{L}$ is the orbital angular momentum and $\vec{S}$ is the spin angular momentum, then $\vec{L}\cdot\vec{S}$ does not commute with **[GATE 2014]**

(a) S_z (b) L^2 (c) S^2 (d) $\left(\vec{L}+\vec{S}\right)^2$

20. ψ_1 and ψ_2 are two orthogonal states of a spin ½ system. it is given that

$$\psi_1 = \frac{1}{\sqrt{3}}\begin{pmatrix}1\\0\end{pmatrix} + \sqrt{\frac{2}{3}}\begin{pmatrix}0\\1\end{pmatrix}$$ **[GATE 2014]**

where $\begin{pmatrix}1\\0\end{pmatrix}$ and $\begin{pmatrix}0\\1\end{pmatrix}$ represents the spin-up and spin-down states, respectively. When the system is in the state ψ_2, its probability to be in the spin-up state is ___________________

21. If L_+ and L_- are the angular momentum ladder operators, then the expectation value of $\left(L_+L_- + L_-L_+\right)$ in the state $|l = 1, m = 1\rangle$ of an atom is ____________ $\hbar^2$. **[GATE 2014]**

22. Let $\vec{L}$ and $\vec{p}$ be the angular and linear momentum operators respectively, for a particle. The commutator $\left[L_x, p_y\right]$ gives **[GATE 2015]**

(a) $-i\hbar p_x$ (b) 0 (c) $i\hbar p_x$ (d) $i\hbar p_z$

23. An operator for a spin-1/2 particle is given by $\hat{A} = \lambda\vec{\sigma}.\vec{B}$, where $\vec{B} = \frac{B}{\sqrt{2}}\left(\hat{x}+\hat{y}\right)$, $\vec{\sigma}$ denotes Pauli spin matrices and λ is a constant. The eigenvalue of $\hat{A}$ are **[GATE 2015]**

(a) $\pm\frac{\lambda B}{\sqrt{2}}$ (b) $\pm\lambda B$ (c) $0, \lambda B$ (d) $0, -\lambda B$

24. σ_x, σ_y and σ_z are the Pauli matrices. The expression $2\sigma_x\sigma_y + \sigma_y\sigma_x$ is equal to **[GATE 2016]**

(a) $-3i\sigma_z$ (b) $-i\sigma_z$ (c) $i\sigma_z$ (d) $3i\sigma_z$

25. Let $|\ell, m\rangle$ be the simultaneous eigenstates of L^2 and L_z. Here $\vec{L}$ is the angular momentum operator with Cartesian components $\left(L_x, L_y, L_z\right)$, ℓ is the angular momentum quantum number and *m* is the azimuthal quantum number. The value of $\langle 1, 0|\left(L_x + iL_y\right)|1, -1\rangle$ is **[GATE 2016]**

(a) 0 (b) $\hbar$ (c) $\sqrt{2}\,\hbar$ (d) $\sqrt{3}\,\hbar$

26. For the Hamiltonian $H = a_0 I + \vec{b}.\vec{\sigma}$ where $a_0 \in R, \vec{b}$ is a real vector, I is the 2×2 identity matrix and $\vec{\sigma}$ are the Pauli matrices, the ground state energy is **[GATE 2017]**

(a) $|b|$ (b) $2a_0 - |b|$ (c) $a_0 - |b|$ (d) a_0

27. The wavefunction of which orbital is spherically symmetric: **[GATE 2017]**

(a) p_x (b) p_y (c) s (d) d_{xy}

28. The electronic ground state energy of the Hydrogen atom is –13.6 eV. The highest possible electronic energy eigenstate has an energy equal to **[GATE 2017]**

(a) 0 (b) 1 eV (c) +13.6 eV (d) ∞

29. Electrons with spin in the z-direction $(\hat{z})$ are passed through a Stern-Gerlach (SG) set up with the magnetic field at $\theta = 60^\circ$ from $\hat{z}$. The fraction of electrons that will emerge with their spin parallel to the magnetic field in the SG set up (rounded off to two decimal places) _________________ **[GATE 2019]**

$$\left[\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\right]$$

ANSWER KEY

1. (d)	**2. (c)**	**3. (c)**	**4. (a)**	**5. (c)**	**6. (b)**	**7. (d)**
8. (b)	**9. (c)**	**10. (c)**	**11. (a)**	**12. (c)**	**13. (a)**	**14. (a)**
15. (a)	**16. (b)**	**17. (d)**	**18. (7.6)**	**19. (a)**	**20. (0.67)**	**21. (2)**
22. (d)	**23. (b)**	**24. (c)**	**25. (c)**	**26. (c)**	**27. (c)**	**28. (a)**
29. (0.75)						

CSIR-UGC-NET Previous Years' Questions

1. The Hamiltonian of an electron in a constant magnetic field $\vec{B}$ is given by $H = \mu\vec{\sigma}.\vec{B}$ where μ is a positive constant and $\vec{\sigma} = (\sigma_1, \sigma_2, \sigma_3)$ denotes the Pauli matrices. Let $\omega = \mu B/\hbar$ and I be the 2×2 unit matrix. Then the operator $e^{iHt/\hbar}$ simplifies to **[CSIR June 2011]**

(a) $I\cos\frac{\omega t}{2} + \frac{i\vec{\sigma}.\vec{B}}{B}\sin\frac{\omega t}{2}$ (b) $I\cos\omega t + \frac{i\vec{\sigma}.\vec{B}}{B}\sin\omega t$

(c) $I\sin\omega t + \frac{i\vec{\sigma}.\vec{B}}{B}\cos\omega t$ (d) $I\sin 2\omega t + \frac{i\vec{\sigma}.\vec{B}}{B}\cos 2\omega t$

2. The component along an arbitrary direction $\hat{n}$, with direction cosines (n_x, n_y, n_z), of the spin of a spin-½ particle is measured. The result is: **[CSIR June 2012]**

(a) 0 (b) $\pm\frac{\hbar}{2}n_z$ (c) $\pm\frac{\hbar}{2}(n_x, n_y, n_z)$ (d) $\pm\frac{\hbar}{2}$

3. In a basis in which the z-component S_z of the spin is diagonal, an electron is in a spin state

$$\psi = \begin{pmatrix} (1+i)/\sqrt{6} \\ \sqrt{2/3} \end{pmatrix}$$

The probabilities that a measurement of S_z will yield the values $\hbar/2$ and $-\hbar/2$ are, respectively **[CSIR June 2013]**

(a) 1/2 and 1/2 (b) 2/3 and 1/3 (c) 1/4 and 3/4 (d) 1/3 and 2/3

4. A spin $-\frac{1}{2}$ particle is in the state $\chi = \frac{1}{\sqrt{11}}\begin{pmatrix} 1+i \\ 3 \end{pmatrix}$ in the eigenbasis of S^2 and S_z. If we measure S_z the probabilities of getting $+\frac{\hbar}{2}$ and $-\frac{\hbar}{2}$, respectively, are **[CSIR Dec 2013]**

(a) $\frac{1}{2}$ and $\frac{1}{2}$ (b) $\frac{2}{11}$ and $\frac{9}{11}$ (c) 0 and 1 (d) $\frac{1}{11}$ and $\frac{3}{11}$

5. Let ψ_{nlm} denote the eigenfunctions of a Hamiltonian for a spherically symmetric potential $V(r)$. The expectation value of L_z in the state

$\psi = \frac{1}{6}\left[\psi_{200} + \sqrt{5}\,\psi_{210} + \sqrt{10}\,\psi_{21-1} + \sqrt{20}\,\psi_{211}\right]$ is **[CSIR Dec 2013]**

(a) $-\frac{5}{18}\hbar$ (b) $\frac{5}{6}\hbar$ (c) $\hbar$ (d) $\frac{5}{18}\hbar$

6. The expectation value of the x-component of the orbital angular momentum L_x in the state $\psi = \frac{1}{5}\left[3\psi_{2,1,-1} + \sqrt{5}\psi_{2,1,0} - \sqrt{11}\psi_{2,1,+1}\right]$ (where ψ_{nlm} are the eigenfunctions in usual notation), is

[CSIR Dec 2013]

(a) $-\frac{\hbar\sqrt{10}}{25}(\sqrt{11}-3)$ (b) 0 (c) $\frac{\hbar\sqrt{10}}{25}(\sqrt{11}+3)$ (d) $\hbar\sqrt{2}$

7. A particle is prepared in a simultaneous eigenstate of L^2 and L_z. If $\ell(\ell+1)\hbar^2$ and $m\hbar$ are respectively the eigenvalues of L^2 and L_z, then the expectation value $\langle L_x^2 \rangle$ of the particle in this state satisfies

[CSIR Dec 2013]

(a) $\langle L_x^2 \rangle = 0$

(b) $0 \le \langle L_x^2 \rangle \le \ell^2\hbar^2$

(c) $0 \le \langle L_x^2 \rangle \le \frac{\ell(\ell+1)\hbar^2}{3}$

(d) $\frac{\ell\hbar^2}{2} \le \langle L_x^2 \rangle \le \frac{\ell(\ell+1)\hbar^2}{2}$

8. Consider the normalized wavefunction $\phi = a_1\psi_{11} + a_2\psi_{10} + a_3\psi_{1-1}$ where ψ_{lm} is a simultaneous normalized eigenfunction of the angular momentum operators L^2 and L_z, with eigenvalues $l(l+1)\hbar^2$ and $m\hbar$ respectively. If ϕ is an eigenfunction of the operator L_x with eigenvalue $\hbar$, then

[CSIR Dec 2014]

(a) $a_1 = -a_3 = \frac{1}{2}, a_2 = \frac{1}{\sqrt{2}}$

(b) $a_1 = a_3 = \frac{1}{2}, a_2 = \frac{1}{\sqrt{2}}$

(c) $a_1 = a_3 = \frac{1}{2}, a_2 = -\frac{1}{\sqrt{2}}$

(d) $a_1 = a_2 = a_3 = \frac{1}{\sqrt{3}}$

9. Let $\vec{\sigma} = (\sigma_1, \sigma_2, \sigma_3)$, where $\sigma_1, \sigma_2, \sigma_3$ are the Pauli matrices. If $\vec{a}$ and $\vec{b}$ are two arbitrary constant vectors in three dimensions, the commutator $[\vec{a}\cdot\vec{\sigma}, \vec{b}\cdot\vec{\sigma}]$ is equal to (in the following *I* is the identity matrix)

[CSIR Dec 2014]

(a) $(\vec{a}\cdot\vec{b})(\sigma_1 + \sigma_2 + \sigma_3)$

(b) $2i(\vec{a}\times\vec{b})\cdot\vec{\sigma}$

(c) $(\vec{a}\cdot\vec{b})I$

(d) $|\vec{a}||\vec{b}|I$

10. If L_i are the components of the angular momentum operator $\vec{L}$, then the operator $\sum i = 1,2,3\left[\left[\vec{L}, L_i\right], L_i\right]$ equals **[CSIR June 2015]**

(a) $\vec{L}$ (b) $2\vec{L}$ (c) $3\vec{L}$ (d) $-\vec{L}$

11. Let ψ_{nlm} denote the eigenstates of a hydrogen atom in the usual notation. The state **[CSIR Dec. 2015]**

$\frac{1}{5}[2\psi_{200} - 3\psi_{211} + \sqrt{7}\psi_{210} - \sqrt{5}\psi_{21-1}]$ is an eigenstate of

(a) L^2, but not of the Hamiltonian or L_z

(b) the Hamiltonian, but not of L^2 or L_z

(c) the Hamiltonian, L^2 and L_z

(d) L^2 and L_z, but not of the Hamiltonian

12. The Hamiltonian for a spin-$\frac{1}{2}$ particle at rest is given by $H = E_0(\sigma_z + \alpha\sigma_x)$, where σ_x and σ_z are Pauli spin matrices and E_0 and α are constants. The eigenvalues of this Hamiltonian are **[CSIR Dec. 2015]**

(a) $\pm E_0\sqrt{1+\alpha^2}$ (b) $\pm E_0\sqrt{1-\alpha^2}$ (c) E_0 (doubly degenerate) (d) $E_0\left(1\pm\frac{1}{2}\alpha^2\right)$

13. The product of the uncertainties $(\Delta L_x)(\Delta L_y)$ for a particle in the state $a|1,1\rangle + b|1,-1\rangle$ (where $|l,m\rangle$ denotes an eigenstate of L^2 and L_z) will be a minimum for **[CSIR Dec. 2015]**

(a) $a = \pm ib$ (b) $a = 0$ and $b = 1$ (c) $a = \frac{\sqrt{3}}{2}$ and $b = \frac{1}{2}$ (d) $a = \pm b$

14. If $\hat{L}_x$, $\hat{L}_y$ and $\hat{L}_z$ are the components of the angular momentum operator in three dimensions, the commutator $\left[\hat{L}_x, \hat{L}_x\hat{L}_y\hat{L}_z\right]$ may be simplified to **[CSIR June 2016]**

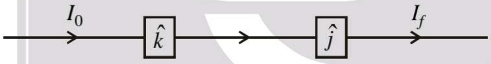
(a) $i\hbar L_x\left(\hat{L}_z^2 - \hat{L}_y^2\right)$ (b) $i\hbar\hat{L}_z\hat{L}_y\hat{L}_x$ (c) $i\hbar L_x\left(2\hat{L}_z^2 - \hat{L}_y^2\right)$ (d) 0

15. The Hamiltonian of a spin-$\frac{1}{2}$ particle in a magnetic field $\vec{B}$ is given by $H = -\mu\vec{B}\cdot\vec{\sigma}$, where μ is a real constant and $\vec{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ are the Pauli spin matrices. If $\vec{B} = (B_0, B_0, 0)$ and the spin state at time $t = 0$ is an eigenstate of σ_x, then of the expectation values $\langle\sigma_x\rangle$, $\langle\sigma_y\rangle$ and $\langle\sigma_z\rangle$ **[CSIR June 2018]**

(a) Only $\langle\sigma_x\rangle$ changes with time (b) Only $\langle\sigma_y\rangle$ changes with time

(c) Only $\langle\sigma_z\rangle$ changes with time (d) All three change with time

16. Two Stern-Gerlach apparatus S_1 and S_2 are kept in a line (x-axis). The directions of their magnetic fields are along the positive z- and y-axes, respectively. Each apparatus only transmits particles with spins aligned in the direction of its magnetic field. If an initially unpolarized beam of spin-$\frac{1}{2}$ particles passes through this configuration, the ratio of intensities $I_0 : I_f$ of the initial and final beams, is **[CSIR June 2018]**



(a) 16 : 1 (b) 2 : 1 (c) 4 : 1 (d) 1 : 0

17. Consider the operator $A_x = L_y p_z - L_z p_y$, where L_i and p_i denote, respectively, the components of the angular momentum and momentum operators. The commutator $[A_x, x]$, where x is the x-component of the position operator, is **[CSIR Dec. 2018]**

(a) $-i\hbar(zp_z + yp_y)$ (b) $-i\hbar(zp_z - yp_y)$ (c) $i\hbar(zp_z + yp_y)$ (d) $i\hbar(zp_z - yp_y)$

18. A system of spin-$\frac{1}{2}$ particles is prepared to be in the eigenstate of σ_z with eigenvalue $+1$. The system is rotated by an angle of 60° about the x-axis. After the rotation, the fraction of the particles that will be measured to be in the eigenstate of σ_z with eigenvalue $+1$ is **[CSIR Dec. 2018]**

(a) 1/3 (b) 2/3 (c) 1/4 (d) 3/4

19. A particle moving in a central potential is described by a wavefunction $\psi(\boldsymbol{r}) = zf(r)$, where $\boldsymbol{r} = (x, y, z)$ is the position vector of the particle and $f(r)$ is a function of $r = |\boldsymbol{r}|$. If $\boldsymbol{L}$ is the total angular momentum of the particle, the value of L^2 must be **[CSIR June 2019]**

(a) $2\hbar^2$ (b) $\hbar^2$ (c) $4\hbar^2$ (d) $\frac{3}{4}\hbar^2$

20. The Hamiltonian of a spin-$\frac{1}{2}$ particle, precessing in the presence of a magnetic field B along the positive x-direction, is $H = -\mu_B B S_x$, where μ_B is the Bohr magneton. At time $t = 0$, the particle is in an eigenstate of S_z with eigenvalue $\hbar/2$. The earliest time when it will be in the eigenstate of S_y with eigenvalue $\hbar/2$, is

(a) $\frac{\pi}{(2\mu_B B)}$ (b) $\frac{\pi}{(4\mu_B B)}$ (c) $\frac{3\pi}{(2\mu_B B)}$ (d) $\frac{2\pi}{(3\mu_B B)}$ **[CSIR Dec. 2019]**

ANSWER KEY

1. (b)	**2. (d)**	**3. (d)**	**4. (b)**	**5. (d)**	**6. (a)**	**7. (d)**
8. (b)	**9. (b)**	**10. (b)**	**11. (b)**	**12. (a)**	**13. (d)**	**14. (a)**
15. (c)	**16. (c)**	**17. (a)**	**18. (d)**	**19. (a)**	**20. (a)**	

JEST Previous Years' Questions

1. Define $\sigma_x = \left(f^\dagger + f\right)$ and $\sigma_y = -i\left(f^\dagger - f\right)$, where the σ' are Pauli spin matrices and $f, f^\dagger$ obey anticommutation relations $\{f, f\} = 0, \{f, f^\dagger\} = 1$. Then σ_z is given by **[JEST 2012]**

   (a) $f^\dagger f - 1$ (b) $2f^\dagger f - 1$ (c) $2f^\dagger f + 1$ (d) $f^\dagger f$

2. Consider a spin-1/2 particle in the presence of homogeneous magnetic field of magnitude B along z-axis which is prepared initially in a state $|\psi\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle + |\downarrow\rangle\right)$ at time $t = 0$. At what time 't' will the particles be in the state $-|\psi\rangle$ (μ_B is Bohr magneton)? **[JEST 2012]**

   (a) $t = \frac{\pi\hbar}{\mu_B B}$ (b) $t = \frac{2\pi\hbar}{\mu_B B}$ (c) $t = \frac{\pi\hbar}{2\mu_B B}$ (d) Never

3. The quantum state $\sin x|\uparrow\rangle + \exp(i\phi)\cos x|\downarrow\rangle$, where $\langle\uparrow||\downarrow\rangle = 0$ and x, ϕ are, real, is orthogonal to: **[JEST 2012]**

   (a) $\sin x|\uparrow\rangle$ (b) $\cos x|\uparrow\rangle + \exp(i\phi)\sin x|\downarrow\rangle$

   (c) $-\cos x|\uparrow\rangle + \exp(i\phi)\sin x|\downarrow\rangle$ (d) $-\exp(i\phi)\cos x|\uparrow\rangle + \sin x|\downarrow\rangle$

4. If J_x, J_y, J_z are angular momentum operators, the eigenvalues of the operator $\left(J_x + J_y\right)/\hbar$ are **[JEST 2013]**

   (a) real and discrete with rational spacing (b) real and discrete with irrational spacing

   (c) real and continuous (d) not all real

5. Consider the state $\begin{pmatrix} 1/2 \\ 1/2 \\ 1/\sqrt{2} \end{pmatrix}$ corresponding to the angular momentum $l = 1$ in the L_z basis of states with m $= +1, 0, -1$. If L_z^2 is measured in this state yielding a result 1, what is the state after the measurement? **[JEST 2013]**

   (a) $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ (b) $\begin{pmatrix} 1/\sqrt{3} \\ 0 \\ \sqrt{2/3} \end{pmatrix}$ (c) $\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ (d) $\begin{pmatrix} 1/\sqrt{2} \\ 0 \\ 1/\sqrt{2} \end{pmatrix}$

6. What are the eigenvalues of the operator $H = \vec{\sigma}.\vec{\alpha}$, where $\vec{\sigma}$ are the three Pauli matrices and $\vec{\alpha}$ is a vector? **[JEST 2013]**

   (a) $\alpha_x + \alpha_y$ and α_z (b) $\alpha_x + \alpha_y$ and $i\alpha_z$ (c) $\pm\left(\alpha_x + \alpha_y + \alpha_z\right)$ (d) $\pm|\vec{\alpha}|$

7. Consider an eigenstate of $\vec{L}^2$ and L_z operator denoted by $|\ell, m\rangle$. Let $A = \hat{n}.\vec{L}$ denote an operator, where $\hat{n}$ is a unit vector parametrized in terms of two angles as $(n_x, n_y, n_z) = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$. The width ΔA in $|\ell, m\rangle$ state is: **[JEST 2014]**

(a) $\sqrt{\frac{\ell(\ell+1)-m^2}{2}}\hbar\cos\theta$ (b) $\sqrt{\frac{\ell(\ell+1)-m^2}{2}}\hbar\sin\theta$

(c) $\sqrt{\ell(\ell+1)-m^2}\hbar\sin\theta$ (d) $\sqrt{\ell(\ell+1)-m^2}\hbar\cos\theta$

8. Suppose a spin $\frac{1}{2}$ particle is in the state, $|\psi\rangle = \frac{1}{\sqrt{6}}\begin{bmatrix}1+i\\2\end{bmatrix}$. If S_x (x component of the spin angular momentum operator) is measured what is the probability of getting $+\frac{\hbar}{2}$? **[JEST 2014]**

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $\frac{5}{6}$ (d) $\frac{1}{6}$

9. A spin-1 particle is in a state $|\psi\rangle$ described by the column matrix $(1/\sqrt{10})\{2, \sqrt{2}, 2i\}$ in the S_z basis. What is the probability that a measurement of operator S_z will yield the result $\hbar$ for the state $S_x|\psi\rangle$?

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{4}$ (d) $\frac{1}{6}$ **[JEST 2016]**

10. A spin-$\frac{1}{2}$ particle in a uniform external magnetic field has energy eigenstates $|1\rangle$ and $|2\rangle$. The system is prepared in ket-state $(|1\rangle + |2\rangle)/\sqrt{2}$ at time $t = 0$. It evolves to the state described by the ket $(|1\rangle - |2\rangle)/\sqrt{2}$ in time T. The minimum energy difference between two levels is: **[JEST 2016]**

(a) $\frac{h}{6T}$ (b) $\frac{h}{4T}$ (c) $\frac{h}{2T}$ (d) $\frac{h}{T}$

11. If $Y_{xy} = \frac{1}{\sqrt{2}}(Y_{2,2} - Y_{2,-2})$, where $Y_{l,m}$ are spherical harmonics, then which of the following is true ?

[JEST 2016]

(a) Y_{xy} is an eigenfunction of both L^2 and L_z (b) Y_{xy} is an eigenfunction of L^2 but not L_z

(c) Y_{xy} is an eigenfunction of L_z but not L^2 (d) Y_{xy} is not an eigenfunction of either L^2 or L_z

12. A spin-$\frac{1}{2}$ particle is in a state $(|\uparrow\rangle + |\downarrow\rangle)/\sqrt{2}$, where $|\uparrow\rangle$ and $|\downarrow\rangle$ are the eigenstates of S_z operator. The exceptation value of the spin angular momentum measured along x-direction is: **[JEST 2016]**

(a) $\hbar$ (b) $-\hbar$ (c) 0 (d) $\frac{\hbar}{2}$

13. If the direction with respect to a right-handed cartesian coordinate system of the ket vector $|z, +\rangle$ is (0, 0, 1), then the direction of the ket vector obtained by application of rotations: $\exp(-i\sigma_z\pi/2)\exp(i\sigma_y\pi/4)$,on the ket $|z, +\rangle$ is (σ_y, σ_z are the Pauli matrices): **[JEST 2016]**

(a) $(0, 1, 0)$ (b) $(1, 0, 0)$ (c) $(1, 1, 0)/\sqrt{2}$ (d) $(1, 1, 1)/\sqrt{3}$

14. If $\rho = \left[I + \frac{1}{\sqrt{3}}\left(\sigma_x + \sigma_y + \sigma_z\right)\right]\Big/2$, where σ's are the Pauli matrices and I is the identity matrix, then the trace of ρ^{2017} is **[JEST 2017]**

(a) 2^{2017} (b) 2^{-2017} (c) 1 (d) 1/2

15. For a spin-$\frac{1}{2}$ particle placed in a magnetic field B, the Hamiltonian is $H = -\gamma B S_y = -\omega S_y$, where S_y is the y-component of the spin operator. The state of the system at time $t = 0$ is $|\psi(t=0)\rangle = |+\rangle$, where $S_z|\pm\rangle = \pm\frac{\hbar}{2}|\pm\rangle$. At a later t, if S_z is measured then what is the probability to get a value $-\hbar/2$?

(a) $\cos^2(\omega t)$ (b) $\sin^2(\omega t)$ (c) 0 (d) $\sin^2\left(\frac{\omega t}{2}\right)$ **[JEST 2019]**

ANSWER KEY

1. (b)	**2. (a)**	**3. (c)**	**4. (a)**	**5. (b)**	**6. (d)**	**7. (b)**
8. (c)	**9. (d)**	**10. (c)**	**11. (b)**	**12. (d)**	**13. (b)**	**14. (c)**
15. (d)						

TIFR Previous Years' Questions

1. Consider the Hamiltonian

$$H = f\vec{\sigma}.\vec{x}$$

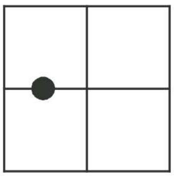
here $\vec{x}$ is the position vector, f is a constant and $\vec{\sigma} = \left(\sigma_x, \sigma_y, \sigma_z\right)$, where $\sigma_x, \sigma_y, \sigma_z$ are the three Pauli matrices. The energy eigenvalues are **[TIFR 2014]**

(a) $f\left(\sqrt{x^2+y^2} \pm z\right)$ (b) $f\left(x \pm iy\right)$ (c) $\pm f\sqrt{x^2+y^2+z^2}$ (d) $\pm f\left(x+y+z\right)$

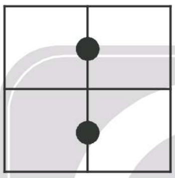
2. In a Stern-Gerlach experiment with spin-½ particles, the beam is found to form two spots on the screen, one directly above the other. The experimenter now makes a hole in the screen at the position of the upper spot. The particles that go through this hole are then passed through another Stern-Gerlach apparatus but with its magnets rotated by 90 degrees counter-clockwise about the axis of the beam direction. Which of the following shows what happens on the second screen?

[TIFR 2015]

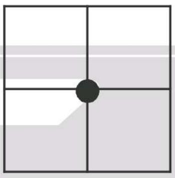
(a)



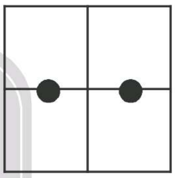
(b)



(c)



(d)



3. A rigid rotator has the wave function **[TIFR 2015]**

$$\psi(\theta,\varphi) = N\left[2i\,Y_{1,0}(\theta,\varphi) + (2+i)Y_{2,-1}(\theta,\varphi) + 3i\,Y_{1,1}(\theta,\varphi)\right]$$

where $Y_{l,m}(\theta,\varphi)$ are the spherical harmonics, and N is a normalization constant. If $\vec{L}$ is the orbital angular momentum operator, and $L_{\pm} = L_x \pm iL_y$ the expectation value of L_+L_- is

(a) $\dfrac{21\hbar^2}{9}$ (b) $\dfrac{23\hbar^2}{9}$ (c) $\dfrac{25\hbar^2}{9}$ (d) 0

4. A unitary matrix U is expanded in terms of a Hermitian matrix H, such that $U = e^{i\pi H/2}$. If we know that

$$H = \begin{pmatrix} \frac{1}{2} & 0 & \frac{\sqrt{3}}{2} \\ 0 & 1 & 0 \\ \frac{\sqrt{3}}{2} & 0 & -\frac{1}{2} \end{pmatrix}$$

then U must be **[TIFR 2017]**

(a) $\begin{pmatrix} i & \frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{1}{2} & i & \frac{1}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} & i \end{pmatrix}$ (b) $\begin{pmatrix} \frac{i}{2} & 0 & \frac{i\sqrt{3}}{2} \\ 0 & i & 0 \\ \frac{i\sqrt{3}}{2} & 0 & -\frac{i}{2} \end{pmatrix}$ (c) $\begin{pmatrix} 1 & 0 & \sqrt{3} \\ 0 & 2 & 0 \\ \sqrt{3} & 0 & -1 \end{pmatrix}$ (d) $\begin{pmatrix} 2i & 1 & \frac{\sqrt{3}}{2} \\ 1 & 2i & 0 \\ \frac{\sqrt{3}}{2} & 0 & 2i \end{pmatrix}$

5. The state $|\Psi\rangle$ of a spin-1 particle is given by **[TIFR 2018]**

$$|\Psi\rangle = \frac{1}{\sqrt{3}}\left(|1,-1\rangle + |1,0\rangle \exp\frac{i\pi}{3} + |1,1\rangle \exp\frac{2i\pi}{3}\right)$$

where $|S, M_s\rangle$ denote the spin eigenstates with eigenvalues $\hbar^2 S(S+1)$ and $\hbar M_s$ respectively. Find $\langle S_x\rangle$, i.e. the expectation value of the x-component of the spin.

ANSWER KEY

1. (c) **2. (d)** **3. (b)** **4. (b)** **5.** $\left(\frac{\sqrt{2}\,\hbar}{3}\right)$

GATE Previous Years' Questions

1. The Hamiltonian of the electron in the hydrogen atom is **[GATE 1991]**

$$H = p^2/(2m) - e^2/(4\pi\varepsilon_0 r)$$

where the symbols have their usual meanings. Let $|\psi\rangle$ denote the ground state of H, with $\langle\psi|\psi\rangle = 1$

(a) $|\psi\rangle$ is an eigen state of $p^2/(2m)$ (b) $\langle\psi|r|\psi\rangle$ is zero

(c) $\langle\psi|(1/r)|\psi\rangle$ is infinite (d) $\langle\psi|H|\psi\rangle < 0$

2. The ground state wave function ψ of hydrogen atom in spherical polar co-ordinates has no angular dependence but only radial dependence. Then ψ is an eigen function of **[GATE 1996]**

(a) L_x, L_y and L_z simultaneously (b) L_z but not of L_x and L_y

(c) none of L_x, L_y and L_z (d) p_z

3. The spinless particle moves in a central potential $V(r)$. Which of the following statement is TRUE? **[GATE 2001]**

(a) The kinetic energy and the potential energy of the particle cannot have simultaneous sharp values.
(b) The total energy and the potential energy of the particle can have simultaneous sharp values.
(c) The total energy and the square of the orbital angular momentum about the origin cannot have simultaneous sharp values.
(d) The total energy of the particle can have only discrete eigenvalues.

Common data for Q. 4 and Q.5

A particle is located in a three dimensional cubic well of width L with impenetrable walls **[GATE 2003]**

4. The sum of he energies of the third and the fourth levels is:

(a) $10\pi^2\hbar^2/mL^2$ (b) $10\pi^2\hbar^2/3\,mL^2$ (c) $11\pi^2\hbar^2/2mL^2$ (d) $15\pi^2\hbar^2/2mL^2$

5. The degeneracy of the fourth level is given by

(a) 1 (b) 2 (c) 3 (d) 4

Linked Answer for Q.6 and Q.7

The one-electron states for non-interacting electrons confined in a cubic box of side a are $\varepsilon_0 < \varepsilon_1 < \varepsilon_2 < \varepsilon_3 < \varepsilon_4$ etc. **[GATE 2004]**

6. The energy of the lowest state is:

(a) zero (b) $\frac{\hbar^2\pi^2}{2ma^2}$ (c) $\frac{\hbar^2\pi^2}{ma^2}$ (d) $\frac{3\hbar^2\pi^2}{2ma^2}$

7. The degeneracy (including spin) of the level ε_3 is:

(a) 2 (b) 4 (c) 6 (d) 8

8. The expectation value of the z coordinate (z), in the ground state of the hydrogen atom (wavefunction: $\psi_{100}(r) = Ae^{-r/a_0}$, where A is the normalization constant and a_0 is the Bohr radius), is **[GATE 2005]**

(a) a_0 (b) $\frac{a_0}{2}$ (c) $\frac{a_0}{4}$ (d) 0

9. A particle is moving in a spherically symmetric potential $V(r) = \alpha r^2$, where α is a postive constant. In a stationary state, the expectation value of the kinetic energy of the particle is **[GATE 2006]**

(a) $\langle T\rangle = \langle V\rangle$ (b) $\langle T\rangle = 2\langle V\rangle$ (c) $\langle T\rangle = 3\langle V\rangle$ (d) $\langle T\rangle = 4\langle V\rangle$

10. An atomic state of hydrogen is represented by the following wave function:

$$\psi(r,\theta,\phi)=\frac{1}{\sqrt{2}}\left(\frac{1}{a_0}\right)^{3/2}\left(1-\frac{r}{2a_0}\right)e^{-r/2a_0}\cos\theta$$

where a_0 is a constant. The quantum number of the state are **[GATE 2007]**

(a) $\ell=0, m=0, n=1$ (b) $\ell=1, m=1, n=2$ (c) $\ell=1, m=0, n=2$ (d) $\ell=2, m=0, n=3$

11. Consider the combined system of proton and electron in the hydrogen atom in its (electronic) ground state. Let I denote the quantum number associated with the total angular momentum and let $\langle m\rangle$ denote the magnitude of the expectation value of the net magnetic moment in the state. Which of the following pairs represents a possible state of the system (μ_B is Bohr magneton)? **[GATE 2008]**

(a) $I=0, \langle m\rangle=0$ (b) $I=\frac{1}{2}, \langle m\rangle=1\mu_B$ (c) $I=1, \langle m\rangle\approx 1\mu_B$ (d) $I=0, \langle m\rangle=2\mu_B$

12. The three principal moments of inertia of a methanol (CH_3OH) molecule have the property $I_x=I_y=I$ and $I_z\neq I$. The rotational energy eigenvalues are **[GATE 2010]**

(a) $\frac{\hbar^2}{2I}\ell(\ell+1)+\frac{\hbar^2 m_\ell^2}{2}\left(\frac{1}{I_z}-\frac{1}{I}\right)$ (b) $\frac{\hbar^2}{2I}\ell(\ell+1)$

(c) $\frac{\hbar^2 m_\ell^2}{2}\left(\frac{1}{I_z}-\frac{1}{I}\right)$ (d) $\frac{\hbar^2}{2I}\ell(\ell+1)+\frac{\hbar^2 m_\ell^2}{2}\left(\frac{1}{I_z}+\frac{1}{I}\right)$

13. The normalized ground state wavefunction of a hydrogen atom is given by $\psi(r)=\frac{1}{\sqrt{4\pi}}\frac{2}{a^{3/2}}e^{-r/a}$, where a is the Bohr radius and r is the distance of the electron from the nucleus, located at the origin. The expectation value $\left\langle\frac{1}{r^2}\right\rangle$ is: **[GATE 2011]**

(a) $\frac{8\pi}{a^2}$ (b) $\frac{4\pi}{a^2}$ (c) $\frac{4}{a^2}$ (d) $\frac{2}{a^2}$

14. A particle of mass m is confined in a two dimensional square well potential of dimension a. This potential $V(x,y)$ is given by

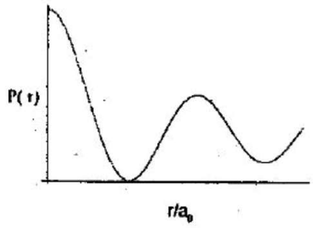
$$V(x,y)=0 \text{ for } 0<x<a \text{ and } 0<y<a$$
$$=\infty \text{ else where}$$

The energy of the first excited state for this particle is given by, **[GATE 2012]**

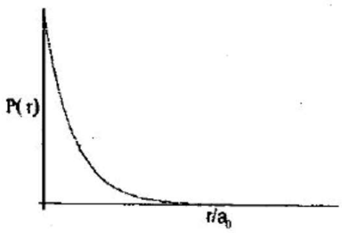
(a) $\frac{\pi^2\hbar^2}{ma^2}$ (b) $\frac{2\pi^2\hbar^2}{ma^2}$ (c) $\frac{5\pi^2\hbar^2}{2ma^2}$ (d) $\frac{4\pi^2\hbar^2}{ma^2}$

15. The ground state wavefunction for the hydrogen atom is given by $\psi_{100} = \frac{1}{\sqrt{4\pi}}\left(\frac{1}{a_0}\right)^{3/2} e^{-r/a_0}$, where a_0 is the Bohr radius. The plot of the radial probability density, P(r) for the hydrogen atom in the ground state is **[GATE 2012]**

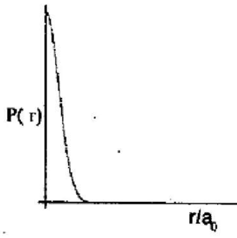
(a)



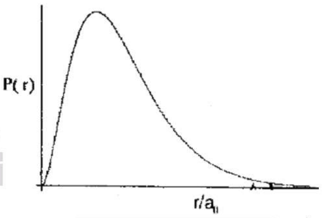
(b)



(c)



(d)



16. An electron in the ground state of the hydrogen atom has the wave function

$$\psi(\vec{r}) = \frac{1}{\sqrt{\pi a_0^3}} e^{-(r/a_0)}$$

where a_0 is constant. The expectation value of the operator $\hat{Q} = z^2 - r^2$, where $z = r\cos\theta$ is (Hint: $\int_0^\infty e^{-ar} r^n dr = \frac{\Gamma(n)}{\alpha^{n+1}} = \frac{(n-1)!}{\alpha^{n+1}}$) **[GATE 2014]**

(a) $-\frac{a_0^2}{2}$ (b) $-a_0^2$ (c) $-\frac{3a_0^2}{2}$ (d) $-2a_0^2$

17. A particle of mass 'm' is subjected to a potential, **[GATE 2014]**

$$V(x, y) = \frac{1}{2}m\omega^2\left(x^2 + y^2\right), \quad -\infty \le x \le \infty, -\infty \le y \le \infty$$

The state with energy $4\hbar\omega$ is g-fold degenerate. The value of g is _________

18. A hydrogen atom is in the state

$$\psi = \sqrt{\frac{8}{21}}\psi_{200} - \sqrt{\frac{3}{7}}\psi_{310} + \sqrt{\frac{4}{21}}\psi_{321}$$ **[GATE 2014]**

where n, ℓ, m in $\psi_{n\ell m}$ denote the principal, orbital and magnetic quantum numbers, respectively. If $\vec{L}$ is the angular momentum operator, the average value of L^2 is _____________ $\hbar^2$.

19. The moment of inertia of a rigid diatomic molecule A is 6 times that of another rigid diatomic molecule B. If the rotational energies of the two molecules are equal, then the corresponding values of the rotational quantum numbers J_A and J_B are **[GATE 2014]**

(a) $J_A = 2, J_B = 1$ (b) $J_A = 3, J_B = 1$ (c) $J_A = 5, J_B = 0$ (d) $J_A = 6, J_B = 1$

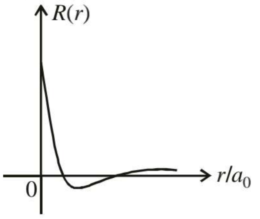
20. Positronium is an atom made of an electron and a positron. Given the Bohr radius for the ground state of the Hydrogen atom to be 0.53 Angstroms, the Bohr radius for the ground state of positronium is ______ Angstroms. (up to two decimal places). **[GATE 2017]**

21. The degeneracy of the third energy level of a 3-dimensional isotropic quantum harmonic oscillator is

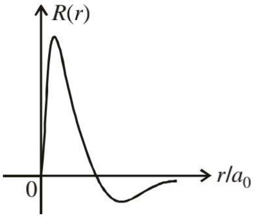
(a) 6 (b) 12 (c) 8 (d) 10 **[GATE 2017]**

22. Which one of the following represents the $3p$ radial wave function of hydrogen atom ? (a_0 is the Bohr radius) **[GATE 2018]**

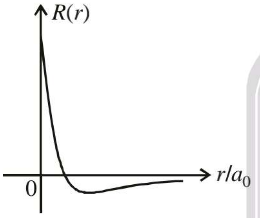
(a)



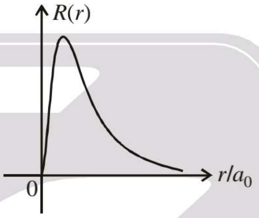
(b)



(c)



(d)



23. The intrinsic/permanent electric dipole moment in the ground state of hydrogen atom is (a_0 is the Bohr radius).

(a) $-3ea_0$ (b) zero (c) ea_0 (d) $3ea_0$ **[GATE 2018]**

24. Given the following table, **[GATE 2018]**

Group-I	**Group-II**
P. Stem-Gerlach experiment	**1.** Wave nature of particles
Q. Zeeman effect	**2.** Quantization of energy of electrons in the atoms
R. Frank-Hertz experiment	**3.** Existence of electron spin
S. Davisson-Germer experiment	**4.** Space quantization of angular momentum

Which one of the following correctly matches the experiments from Group-I to their inferences in Group-II ?

(a) P-2, Q-3, R-4, S-1 (b) P-1, Q-3, R-2, S-4

(c) P-3, Q-4, R-2, S-1 (d) P-2, Q-1, R-4, S-3

25. The Hamiltonian for a quantum harmonic oscillator of mass m in three dimensions is $H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 r^2$ where ω is the angular frequency. The expectation value of r^2 in the first excited state of the oscillator in units of $\frac{\hbar}{m\omega}$ (rounded off to one decimal place) is ________ **[GATE 2019]**

ANSWER KEY

1. (d)	2. (a)	3. (a)	4. (a)	5. (c)	6. (d)	7. (c)
8. (d)	9. (a)	10. (c)	11. (b)	12. (a)	13. (d)	14. (c)
15. (d)	16. (d)	17. (4)	18. (2)	19. (b)	20. (0.99 to 1.10)	
21. (a)	22. (b)	23. (b)	24. (c)	25. (2.5)		

CSIR-UGC-NET Previous Years' Questions

1. The energy levels of the non-relativistic electron in a hydrogen atom (i.e. in a Coulomb potential $V(r) \propto -1/r$ are given by $E_{n\ell m} \propto -1/n^2$, where n is the principal quantum number, and the corresponding wave functions are given by $\psi_{n\ell m}$, where ℓ is the orbital angular momentum quantum number and m is the magnetic quantum number. The spin of the electron is not considered. Which of the following is a correct statement? **[CSIR June 2011]**

(a) There are exactly $(2\ell+1)$ different wave functions $\psi_{n\ell m}$, for each $E_{n\ell m}$.

(b) There are $\ell(\ell+1)$ different wave functions $\psi_{n\ell m}$, for each $E_{n\ell m}$.

(c) $E_{n\ell m}$ does not depend on ℓ and m for the Coulomb potential.

(d) There is a unique wave function $\psi_{n\ell m}$ and $E_{n\ell m}$.

2. If an electron is in the ground state of the hydrogen atom, the probability that its distance from the proton is more than one Bohr radius is approximately **[CSIR June 2011]**

(a) 0.68 (b) 0.48 (c) 0.28 (d) 0.91

3. Given that the ground state energy of the hydrogen atom is -13.6 eV, the ground state energy of positronium (which is a bound state of an electron and a positron) is **[CSIR Dec 2011]**

(a) 6.8 eV (b) -6.8 eV (c) -13.6 eV (d) -27.2 eV

4. The energy of the first excited quantum state of a particle in the two-dimensional potential $V(x,y) = \frac{1}{2}m\omega^2(x^2+4y^2)$ is: **[CSIR Dec 2011]**

(a) $2\hbar\omega$ (b) $3\hbar\omega$ (c) $\frac{3}{2}\hbar\omega$ (d) $\frac{5}{2}\hbar\omega$

5. A particle of mass m is in a cubic box of size a. The potential inside the box $(0 \le x < a, 0 \le y < a, 0 \le z < a)$ is zero and infinite outside. If the particle is in an eigenstate of energy $E = \frac{14\pi^2 h^2}{2ma^2}$, its wave function is: **[CSIR June 2012]**

(a) $\psi = \left(\frac{2}{a}\right)^{3/2} \sin\frac{3\pi x}{a} \sin\frac{5\pi y}{a} \sin\frac{6\pi z}{a}$ (b) $\psi = \left(\frac{2}{a}\right)^{3/2} \sin\frac{7\pi x}{a} \sin\frac{4\pi y}{a} \sin\frac{3\pi z}{a}$

(c) $\psi = \left(\frac{2}{a}\right)^{3/2} \sin\frac{4\pi x}{a} \sin\frac{8\pi y}{a} \sin\frac{2\pi z}{a}$ (d) $\psi = \left(\frac{2}{a}\right)^{3/2} \sin\frac{\pi x}{a} \sin\frac{2\pi y}{a} \sin\frac{3\pi z}{a}$

6. Let $\Psi_{n\ell m}$ denote the eigenfunctions of a Hamiltonian for a spherically symmetric potential V(r). The wavefunction $\Psi = \frac{1}{4}\left[\Psi_{210} + \sqrt{5}\Psi_{21-1} + \sqrt{10}\Psi_{213}\right]$ is an eigenfunction of **[CSIR June 2012]**

(a) H, L^2 and L_z (b) H and L_z (c) H and L^2 (d) L^2 and L_z

7. The wave function of a state of the hydrogen atom is given by

$$\psi = \psi_{200} + 2\psi_{211} + 3\psi_{210} + \sqrt{2}\,\psi_{21-1}$$

where $\psi_{n\ell m}$ denotes the normalized eigen function of the state with quantum numbers n, l and m in the usual notation. The expectation value of L_z in the state ψ is: **[CSIR Dec 2012]**

(a) $\frac{15\hbar}{16}$ (b) $\frac{11\hbar}{16}$ (c) $\frac{3\hbar}{8}$ (d) $\frac{\hbar}{8}$

8. The un-normalized wavefunction of a particle in a spherically symmetric potential is given by $\psi(\vec{r}) = zf(r)$ where f(r) is a function of the radial variable r. The eigenvalue of the operator $\vec{L}^2$ (namely the square of the orbital angular momentum) is **[CSIR June 2013]**

(a) $\hbar^2/4$ (b) $\hbar^2/2$ (c) $\hbar^2$ (d) $2\hbar^2$

9. If ψ_{nlm} denotes the eigenfunction of the Hamiltonian with a potential V = V(r) then the expectation value of the operator $L_x^2 + L_y^2$ in the state $\psi = \frac{1}{5}[3\psi_{211} + \psi_{210} - \sqrt{15}\psi_{21-1}]$ is

[CSIR June 2013]

(a) $39\hbar^2/25$ (b) $13\hbar^2/25$ (c) $2\hbar^2$ (d) $26\hbar^2/25$

10. A particle of mass m in three dimensions is in the potential

$$V(r) = \begin{cases} 0 & r < a \\ \infty & r \geq a \end{cases}$$

Its ground state energy is **[CSIR June 2014]**

(a) $\frac{\pi^2\hbar^2}{2ma^2}$ (b) $\frac{\pi^2\hbar^2}{ma^2}$ (c) $\frac{3\pi^2\hbar^2}{2ma^2}$ (d) $\frac{9\pi^2\hbar^2}{2ma^2}$

11. Given that $\hat{p}_r = -i\hbar\left(\frac{\partial}{\partial r} + \frac{1}{r}\right)$, the uncertainty Δp_r in the ground state

$$\psi_0(r) = \frac{1}{\sqrt{\pi a_0^3}} e^{-r/a_0}$$

of the hydrogen atom is **[CSIR June 2014]**

(a) $\frac{\hbar}{a_0}$ (b) $\frac{\sqrt{2}\hbar}{a_0}$ (c) $\frac{\hbar}{2a_0}$ (d) $\frac{2\hbar}{a_0}$

12. A particle of mass m in the potential $V(x,y) = \frac{1}{2}m\omega^2(4x^2 + y^2)$, is in an eigenstate of energy $E = \frac{5}{2}\hbar\omega$. The corresponding un-normalized eigenfunction is **[CSIR June 2014]**

(a) $y\exp\left[-\frac{m\omega}{2\hbar}(2x^2 + y^2)\right]$ (b) $x\exp\left[-\frac{m\omega}{2\hbar}(2x^2 + y^2)\right]$

(c) $y\exp\left[-\frac{m\omega}{2\hbar}(x^2 + y^2)\right]$ (d) $xy\exp\left[-\frac{m\omega}{2\hbar}(x^2 + y^2)\right]$

13. An electron is in the ground state of a hydrogen atom. The probability that it is within the Bohr radius is approximately equal to **[CSIR June 2014]**

(a) 0.60 (b) 0.90 (c) 0.16 (d) 0.32

14. The ratio of the energy of the first excited state E_1, to that of the ground state E_0 of a particle in a three-dimensional rectangular box of sides L, L and $L/2$ is **[CSIR June 2015]**

(a) 3:2 (b) 2:1 (c) 4:1 (d) 4:3

15. A positron is suddenly absorbed by the nucleus of a tritium $({}^3_1\mathrm{H})$ atom to turn the latter into a He$^+$ ion. If the electron in the tritium atom was initially in the ground state, the probability that the resulting He$^+$ ion will be in its ground state is **[CSIR Dec. 2015]**

(a) 1 (b) $\frac{8}{9}$ (c) $\frac{128}{243}$ (d) $\frac{512}{729}$

16. Suppose that the Coulomb potential of the hydrogen atom is changed by adding an inverse-square term such that the total potential is $V(\vec{r}) = -\frac{ze^2}{r} + \frac{g}{r^2}$, where g is a constant. The energy eigenvalues E_{nlm} in the modified potential **[CSIR June 2016]**

(a) depend on n and l, but not on m (b) depend on n but not on l and m
(c) depend on n and m, but not on l (d) depend explicitly on all three quantum numbers n, l and m

17. The normalized wavefunction of a particle in three dimensions is given by **[CSIR Dec. 2017]**

$$\psi(r, \theta, \varphi) = \frac{1}{\sqrt{8\pi a^3}} e^{-r/2a}$$

where $a > 0$ is a constant. The ratio of the most probable distance from the origin to the mean distance from the origin, is

[You may use $\int_0^\infty dx\, x^n e^{-x} = n!$].

(a) $\frac{1}{3}$ (b) $\frac{1}{2}$ (c) $\frac{3}{2}$ (d) $\frac{2}{3}$

18. Consider a set of particles which interact by a pair potential $V = ar^6$, where r is the inter-particle separation and $a > 0$ is a constant. If a system of such particles has reached virial equilibrium, the ratio of the kinetic to the total energy of the system is **[CSIR Dec. 2017]**

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{3}{4}$ (d) $\frac{2}{3}$

19. A particle of mass m is confined in a three-dimensional box by the potential **[CSIR June 2018]**

$$V(x, y, z) = \begin{cases} 0, & 0 \le x, y, z \le a \\ \infty, & \text{otherwise} \end{cases}$$

The number of eigenstates of Hamiltonian with energy $\frac{9\hbar^2\pi^2}{2ma^2}$ is

(a) 1 (b) 6 (c) 3 (d) 4

20. Let the wavefunction of the electron in a hydrogen atom be

$$\psi(\vec{r}) = \frac{1}{\sqrt{6}}\phi_{200}(\vec{r}) + \sqrt{\frac{2}{3}}\phi_{21-1}(\vec{r}) - \frac{1}{\sqrt{6}}\phi_{100}(\vec{r})$$

where $\phi_{nlm}(\vec{r})$ are the eigenstates of the Hamiltonian in the standard notation. The expectation value of the energy in this state is

(a) -10.8 eV (b) -6.2 eV (c) -9.5 eV (d) -5.1 eV

21. The ground state energy of an anisotropic harmonic oscillator described by the potential

$V(x, y, z) = \frac{1}{2}m\omega^2x^2 + 2m\omega^2y^2 + 8m\omega^2z^2$ (in units of $\hbar\omega$) is **[CSIR Dec. 2018]**

(a) 5/2 (b) 7/2 (c) 3/2 (d) 1/2

22. If the position of the electron in the ground state of a hydrogen atom is measured, the probability that it will be found at a distance $r \geq a_0$ (a_0 being Bohr radius) is nearest to **[CSIR Dec. 2018]**

(a) 0.91 (b) 0.66 (c) 0.32 (d) 0.13

23. The normalized wavefunction of a particle in three-dimensions is given by **[CSIR Dec. 2019]**

$$\psi(x, y, z) = N z \exp\left[-a(x^2 + y^2 + z^2)\right]$$

where a is a positive constant and N is a normalization constant. If L is the angular momentum operator, the eigenvalues, of L^2 and L_z, respectively, are

(a) $2\hbar^2$ and $\hbar$ (b) $\hbar^2$ and 0 (c) $2\hbar^2$ and 0 (d) $\frac{3}{4}\hbar^2$ and $\frac{1}{2}\hbar$

24. The wavefunction of a particle of mass m, constrained to move on a circle of unit radius centered at the origin in the xy-plane, is described by $\psi(\phi) = A\cos^2\phi$, where ϕ is the azimuthal angle. All the possible outcomes of measurements of the z-component of the angular momentum L_z in this state, in units of $\hbar$, are

(a) ± 1 and 0 (b) ± 1 (c) ± 2 (d) ± 2 and 0 **[CSIR Dec. 2019]**

25. Consider the degenerate first excited state $n = 2, \ell = 1$ or the hydrogen atom. It is Hamiltonian is modified by adding $H_1 = \frac{\alpha}{\hbar}L_x$ Where L_x is the x-component of the orbital angular momentum, the changes in the energies of the states labelled by $m = 0, +1$ and -1 are, respectively, **[CSIR Dec. 2019]**

(a) $0, \alpha$ and $-\alpha$ (b) $\alpha, 2\alpha$ and 0 (c) $\alpha, 0$ and $-\alpha$ (d) $0, 2\alpha$ and -2α

ANSWER KEY

1. (c)	2. (a)	3. (b)	4. (d)	5. (d)	6. (c)	7. (d)
8. (d)	9. (d)	10. (a)	11. (a)	12. (a)	13. (d)	14. (a)
15. (d)	16. (a)	17. (d)	18. (c)	19. (c)	20. (d)	21. (b)
22. (b)	23. (c)	24. (d)	25. (a)			

JEST Previous Years' Questions

1. Consider a particle of mass 'm' moving inside a two-dimensional square box whose sides are described by the equations $x = 0,\ x = L,\ y = 0,\ y = L$. What is the lowest eigenvalue of an eigenstate which changes sign under the exchange of x and y? **[JEST 2012]**

(a) $\frac{\hbar^2}{mL^2}$ (b) $\frac{3\hbar^2}{2mL^2}$ (c) $\frac{5\hbar^2}{2mL^2}$ (d) $\frac{7\hbar^2}{2mL^2}$

2. The binding energy of the hydrogen atom (electron bound to proton) is 13.6 eV. The binding energy of positronium (electron bound to positron) is **[JEST 2012]**

(a) 13.6/2 eV (b) 13.6/180 eV (c) 13.6×1810 eV (d) 13.6×2 eV

3. If a proton were ten times lighter, then the ground state energy of the electron in a hydrogen atom would have been **[JEST 2013, 2014]**

(a) Less (b) More (c) The same (d) Depends on the electron mass

4. A particle moving under the influence of a potential $V(r) = kr^2/2$ has a wavefunction $\psi(r,t)$. If the wavefunction changes to $\psi(\alpha r, t)$, the ratio of the average final kinetic energy to the initial kinetic energy will be,

(a) $\frac{1}{\alpha^2}$ (b) α (c) $\frac{1}{\alpha}$ (d) α^2 **[JEST 2015]**

5. The wavefunction of a hydrogen atom is given by the following superposition of energy eigen-functions $\psi_{nlm}(\vec{r})$ (n, l, m are the usual quantum numbers); **[JEST 2016]**

$$\psi(\vec{r}) = \frac{\sqrt{2}}{\sqrt{7}}\psi_{100}(\vec{r}) - \frac{3}{\sqrt{14}}\psi_{210}(\vec{r}) + \frac{1}{\sqrt{14}}\psi_{322}(\vec{r})$$

The ratio of expectation value of the energy to the ground state energy and the expectation value of L^2 are, respectively:

(a) $\frac{229}{504}$ and $\frac{12\hbar^2}{7}$ (b) $\frac{101}{504}$ and $\frac{12\hbar^2}{7}$ (c) $\frac{101}{504}$ and $\hbar^2$ (d) $\frac{229}{504}$ and $\hbar^2$

6. The Hamiltonian of a quantum particle of mass m confined to a ring of unit radius is: **[JEST 2016]**

$$H = \frac{\hbar^2}{2m}\left(-i\frac{\partial}{\partial\theta} - \alpha\right)^2,$$

where θ is the angular coordinate, α is a constant. The energy eigenvalues and eigenfunctions of the particle are (n is an integer):

(a) $\psi_n(\theta) = \frac{e^{in\theta}}{\sqrt{2\pi}}$ and $E_n = \frac{\hbar^2}{2m}(n-\alpha)^2$ (b) $\psi_n(\theta) = \frac{\sin(n\theta)}{\sqrt{2\pi}}$ and $E_n = \frac{\hbar^2}{2m}(n-\alpha)^2$

(c) $\psi_n(\theta) = \frac{\cos(n\theta)}{\sqrt{2\pi}}$ and $E_n = \frac{\hbar^2}{2m}(n-\alpha)^2$ (d) $\psi_n(\theta) = \frac{e^{in\theta}}{\sqrt{2\pi}}$ and $E_n = \frac{\hbar^2}{2m}(n+\alpha)^2$

7. In the ground state of hydrogen atom, the most probable distance of the electron from the nucleus, in units of Bohr radius a_0 is: **[JEST 2016]**

(a) $\frac{1}{2}$ (b) 1 (c) 2 (d) $\frac{3}{2}$

8. A quantum particle of mass m is moving on a horizontal circular path of radius a. The particle is prepared in a quantum state described by the wavefunction, **[JEST 2018]**

$$\psi = \sqrt{\frac{4}{3\pi}}\cos^2\phi$$

ϕ being the azimuthal angle. If a measurement of the z-component of orbital angular momentum of the particle is carried out, the possible outcomes and the corresponding probabilities are

(a) $L_z = 0, \pm\hbar, \pm 2\hbar$ with $P(0) = \frac{1}{5}, P(\pm\hbar) = \frac{1}{5}$ and $P(\pm 2\hbar) = \frac{1}{5}$

(b) $L_z = 0$ with $P(0) = 1$

(c) $L_z = 0, \pm\hbar$ with $P(0) = \frac{1}{3}$ and $\text{P}(\pm\hbar) = \frac{1}{3}$

(d) $L_z = 0, \pm 2\hbar$ with $P(0) = \frac{2}{3}$ and $\text{P}(\pm 2\hbar) = \frac{1}{6}$

ANSWER KEY

1. (c)	**2. (a)**	**3. (b)**	**4. (d)**	**5. (a)**	**6. (a)**	**7. (b)**
8. (d)						

TIFR Previous Years' Questions

1. The normalized wavefunctions of a Hydrogen atom are denoted by $\psi_{n,\ell,m}(\vec{x})$, where n, ℓ and m are, respectively, the principal, azimuthal and magnetic quantum numbers respectively. Now consider an electron in the mixed state

$$\Psi(\vec{x}) = \frac{1}{3}\psi_{1,0,0}(\vec{x}) + \frac{2}{3}\psi_{2,1,0}(\vec{x}) + \frac{2}{3}\psi_{3,2,-2}(\vec{x})$$ **[TIFR 2012]**

The expectation value $\langle E \rangle$ of the energy of this electron, in electron-Volts (eV) will be approximately

(a) –1.5 (b) –3.7 (c) –13.6 (d) –80.1 (e) +13.6

2. An energy eigenstate of the Hydrogen atom has the wave function

$$\psi_{n\ell m}(r,\theta,\varphi) = \frac{1}{81\sqrt{\pi}}\left(\frac{1}{a_0}\right)^{3/2} \sin\theta\cos\theta\exp\left[-\left(\frac{r}{3a_0}+i\varphi\right)\right]$$ **[TIFR 2013]**

where a_0 is the Bohr radius. The principal (n), azimuthal (ℓ) and magnetic (m) quantum numbers corresponding to this wave function are

(a) $n=3, \ell=2, m=1$ (b) $n=2, \ell=1, m=1$

(c) $n=3, \ell=2, m=-1$ (d) $n=2, \ell=1, m=\pm 1$

3. The velocity of an electron in the ground state of a hydrogen atom is v_H. If v_p be the velocity of an electron in the ground state of positronium, then **[TIFR 2013]**

(a) $v_p = v_H$ (b) $v_p = 2v_H$ (c) $v_p = \frac{v_H}{2}$ (d) $v_p = \sqrt{2}v_H$ **Ans. (a)**

4. A rigid rotator is in a quantum state described by the wavefunction

$$\psi(\theta,\varphi) = \sqrt{\frac{3}{4\pi}}\sin\theta\sin\varphi$$ **[TIFR 2014]**

where θ and φ are the usual polar angles. If two successive measurements of L_z are made on this rotator, the probability that the second measurement will yield the value $+\hbar$ is

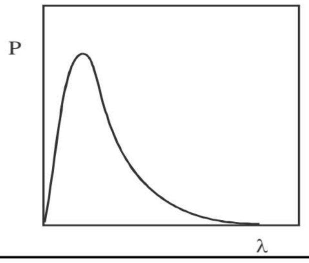
(a) 0.25 (b) 0.33 (c) 0.5 (d) negligible

5. A particle in the 2s state of hydrogen has the wave function

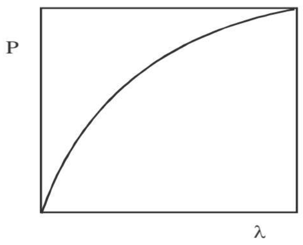
$$\psi_{2s}(r) = \frac{1}{4\sqrt{2}\,\pi}\left(\frac{1}{a_0}\right)^{3/2}\left(2-\frac{r}{a_0}\right)\exp\left(-\frac{r}{2a_0}\right)$$

where 'r' is the radial coordinate w.r.t. the nucleus as origin and a_0 is the Bohr radius. The probability P of finding the electron somewhere inside a sphere of radius λa_0 centred at the nucleus, is best described by the graph **[TIFR 2014]**

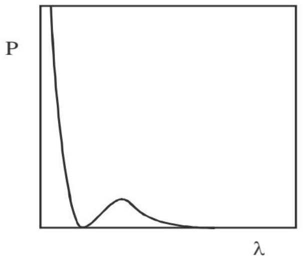
(a)



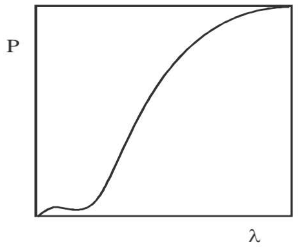
(b)



(c)



(d)



6. The ground state energy of a particle of mass m in a three dimensional cubical box of side ℓ is not zero but $\frac{3h^2}{8m\ell^2}$. This is because **[TIFR 2015]**
   (a) The ground state has no nodes in the interior of the box.
   (b) This is the most convenient choice of the zero level of potential energy.
   (c) Position and momentum cannot be exactly determined simultaneously.
   (d) The potential at the boundaries is not really infinite, but just very large.

7. A quantum mechanical plane rotator consists to two rigidly connected particles of mass m and connected by a massless rod of length d is rotating in the xy-plane about their centre of mass. Suppose that the initial state of the rotor is given by $\psi(\varphi, t=0) = A\cos^2\varphi$, where φ is the angle between one mass and the x-axis, while A is normalization constant. Find the exception value of $3\hat{L}_z^2$ in this state, in units of $\hbar^2$. **[TIFR 2016]**

8. The energy of an electron in the ground state of the He atom is –79 eV. Considering the Bohr model of the atom, what would be 10 times the first ionization potential for a He$^+$ ion, in units of eV ? **[TIFR 2017]**

9. Electrons in a given system of hydrogen atoms are described by the wavefunction

$$\psi(r,\theta,\varphi) = 0.8\,\Psi_{100} + 0.6\,e^{i\pi/3}\,\Psi_{311}$$

where the Ψ_{nlm} denote normalized energy eigenstates. If $(\hat{L}_x, \hat{L}_y, \hat{L}_z)$ are the components of the orbital angular momentum operator, the expectation value of $\hat{L}_x^2$ in this system is **[TIFR 2017]**

(a) $1.5\hbar^2$ (b) $0.36\hbar^2$ (c) $0.18\hbar^2$ (d) zero

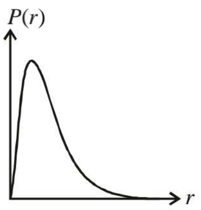
10. A particle is in the ground state of a cubical box of side l. Suddenly one side of the box changes from l to $4l$. If p is the probability of finding the particles in the ground state of the new box, what is $1000p$? **[TIFR 2018]**

11. An electron is in the 2s level of the hydrogen atom, with the radial wave-function

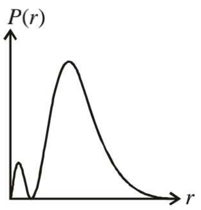
$$\psi(r) = \frac{1}{2\sqrt{2}a_0^{3/2}}\left(2 - \frac{r}{a_0}\right)\exp\left(-\frac{r}{2a_0}\right).$$

The probability $P(r)$ of finding this electron between distances r to $r+dr$ from the centre is best represented by the sketch **[TIFR 2018]**

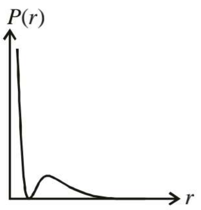
(a)



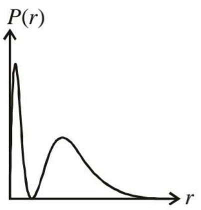
(b)



(c)



(d)



12. An electron in a hydrogen atom is in a state described by the wavefunction:

$$\psi(\vec{r}) = \frac{1}{\sqrt{10}}\psi_{100}(\vec{x}) + \sqrt{\frac{2}{5}}\psi_{210}(\vec{x}) + \sqrt{\frac{2}{5}}\psi_{211}(\vec{x}) - \frac{1}{\sqrt{10}}\psi_{21,-1}(\vec{x})$$

where $\psi_{nlm}(\vec{x})$ denotes a normalized wavefunction of the hydrogen atom with the principal quantum number n, angular quantum number l and magnetic quantum number m. **[TIFR 2019]**

Neglecting the spin-orbit interaction, the expectation values of $\hat{L}_z$ and $\hat{L}^2$ for this state are

(a) $\frac{3\hbar}{4}, \frac{9\hbar^2}{25}$ (b) $\frac{3\hbar}{10}, \frac{9\hbar^2}{5}$ (c) $\frac{3\hbar}{5}, \frac{9\hbar^2}{10}$ (d) $\frac{8\hbar}{10}, \frac{3\hbar^2}{5}$

ANSWER KEY

1. (b) 2. (c) 3. (a) 4. (c) 5. (d) 6. (c) 7. (004)

8. (246) 9. (c) 10. (058) 11. (b) 12. (b)

GATE Previous Years' Questions

1. Which one of the following is not a Boson? **[GATE 1994]**

(a) ${}^{4}_{2}\mathrm{He}$ (b) ${}^{1}_{1}\mathrm{H}$ (c) ${}^{3}_{2}\mathrm{He}$ (d) photon

2. The energy of a single particle in an infinite potential box is $E_0 n^2$ where $E_0 = \hbar^2\pi^2/2ma^2,\ n = 1, 2, 3, ...$ If three non-interacting bosons are in such a potential box, the ground state energy of the system will be

(a) $3E_0$ (b) $4E_0$ (c) $6E_0$ (d) $9E_0$ **[GATE 1998]**

3. Two spin -1/2 fermions having spins $\vec{S}_1$ and $\vec{S}_2$ interact via a potential $V(r) = \vec{S}_1 \cdot \vec{S}_2 V_0(r)$. The contributions of this potential in the singlet and triplet states, respectively, are **[GATE 2004]**

(a) $-\frac{3}{2}V_0(r)$ and $\frac{1}{2}V_0(r)$ (b) $\frac{1}{2}V_0(r)$ and $-\frac{3}{2}V_0(r)$

(c) $\frac{1}{4}V_0(r)$ and $-\frac{3}{4}V_0(r)$ (d) $-\frac{3}{4}V_0(r)$ and $\frac{1}{4}V_0(r)$

4. The wavefunctions of two identical particles in state n and s are given by $\phi_n(r_1)$ and $\phi_s(r_2)$, respectively. The particles obey Maxwell-Boltzmann statistics. The state of the combined two-particle system is expressed as **[GATE 2006]**

(a) $\phi_n(r_1) + \phi_s(r_2)$ (b) $\frac{1}{\sqrt{2}}\left[\phi_n(r_1)\phi_s(r_2) + \phi_n(r_2)\phi_s(r_1)\right]$

(c) $\frac{1}{\sqrt{2}}\left[\phi_n(r_1)\phi_s(r_2) - \phi_n(r_2)\phi_s(r_1)\right]$ (d) $\phi_n(r_1)\phi_s(r_2)$

5. Consider the wavefunction $\psi = \psi(\vec{r}_1, \vec{r}_2)\chi_s$ for a fermionic system consisting of two spin-half particles. The spatial part of the wavefunction is given by **[GATE 2012]**

$$\psi(\vec{r}_1, \vec{r}_2) = \frac{1}{\sqrt{2}}\left[\phi_1(\vec{r}_2)\phi_2(\vec{r}_2) + \phi_2(\vec{r}_1)\phi_1(\vec{r}_2)\right]$$

Where ϕ_1 and ϕ_2 are single particle states. The spin part χ_s of the wavefunction with spin states $\alpha\left(+\frac{1}{2}\right)$ and $\beta\left(-\frac{1}{2}\right)$ should be

(a) $\frac{1}{\sqrt{2}}(\alpha\beta + \beta\alpha)$ (b) $\frac{1}{\sqrt{2}}(\alpha\beta - \beta\alpha)$ (c) $\alpha\alpha$ (d) $\beta\beta$

6. The ground state and the first excited state wave functions of a one dimensional infinite potential well are ψ_1 and ψ_2, respectively. When two spin-up electrons are placed in this potential, which one of the following, with x_1 and x_2 denoting the position of the two electrons, correctly represents the space part of the ground state wave function of the system? **[GATE 2014]**

(a) $\frac{1}{\sqrt{2}}\left[\psi_1(x_1)\psi_2(x_1) - \psi_1(x_2)\psi_2(x_2)\right]$ (b) $\frac{1}{\sqrt{2}}\left[\psi_1(x_1)\psi_2(x_2) + \psi_1(x_2)\psi_2(x_1)\right]$

(c) $\frac{1}{\sqrt{2}}\left[\psi_1(x_1)\psi_2(x_1) + \psi_1(x_2)\psi_2(x_2)\right]$ (d) $\frac{1}{\sqrt{2}}\left[\psi_1(x_1)\psi_2(x_2) - \psi_1(x_2)\psi_2(x_1)\right]$

7. Which one of the following is a fermion? **[GATE 2014]**

(a) $\alpha-$particle (b) ${}_4Be^7$ nucleus (c) Hydrogen atom (d) Deuteron

8. The Pauli matrices for three spin-$\frac{1}{2}$ particles are $\vec{\sigma}_1, \vec{\sigma}_2$ and $\vec{\sigma}_3$, respectively. The dimension of the Hilbert space required to define an operator $\hat{O} = \vec{\sigma}_1 \cdot \vec{\sigma}_2 \times \vec{\sigma}_3$ is_______ **[GATE 2015]**

9. Let the Hamiltonian for two spin-$\frac{1}{2}$ particles of equal masses m, momenta $\vec{p}_1$ and $\vec{p}_2$ and positions $\vec{r}_1$ and $\vec{r}_2$ be $H = \frac{1}{2m}p_1^2 + \frac{1}{2m}p_2^2 + \frac{1}{2}m\omega^2\left(r_1^2 + r_2^2\right) + k\vec{\sigma}_1 \cdot \vec{\sigma}_2$, where $\vec{\sigma}_1$ and $\vec{\sigma}_2$ denote the corresponding Pauli matrices, $\hbar\omega = 0.1eV$ and $k = 0.2eV$. If the ground state has net spin zero, then the energy (in eV) is_______ **[GATE 2015]**

10. Consider a system of eight non-interacting, identical quantum particles of spin-$\frac{3}{2}$ in a one dimensional box of length L. The minimum excitation energy of the system, in units of $\frac{\pi^2\hbar^2}{2mL^2}$ is_______ **[GATE 2015]**

11. If $\vec{s}_1$ and $\vec{s}_2$ are the spin operators of the two electrons of a He atom, the value of $\langle \vec{s}_1 \cdot \vec{s}_2 \rangle$ for the ground state is **[GATE 2016]**

(a) $-\frac{3}{2}\hbar^2$ (b) $-\frac{3}{4}\hbar^2$ (c) 0 (d) $\frac{1}{4}\hbar^2$

12. A two-dimensional square rigid box of side L contains six non-interacting electrons at $T = 0$ K. The mass of the electron is m. The ground state energy of the system of electrons, in units of $\frac{\pi^2\hbar^2}{2mL^2}$ is _______. **[GATE 2016]**

13. For a spin-½ particle, let $|\uparrow\rangle$ and $|\downarrow\rangle$ denote its spin up and spin down states, respectively. If $|a\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle|\downarrow\rangle + |\downarrow\rangle|\uparrow\rangle\right)$ and $|b\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle|\downarrow\rangle - |\downarrow\rangle|\uparrow\rangle\right)$ are composite states of two such particles, which of the following statements is true for their total spin S? **[GATE 2019]**

(a) S = 1 for $|a\rangle$ and $|b\rangle$ is not an eigenstate of the operator $\hat{S}^2$

(b) $|a\rangle$ is not an eigenstate of the operator, $\hat{S}^2$ and $S = 0$ for $|b\rangle$

(c) $S = 0$ for $|a\rangle$ and $S = 1$ for $|b\rangle$

(d) $S = 1$ for $|a\rangle$, and $S = 0$ for $|b\rangle$

ANSWER KEY

1. (c) **2. (a)** **3. (d)** **4. (d)** **5. (b)** **6. (d)** **7. (b)**

8. (8) **9. (–0.3)** **10. (5)** **11. (b)** **12. (24)** **13. (d)**

CSIR-UGC-NET Previous Years' Questions

1. In a system consisting of two spin $\frac{1}{2}$ particles labeled 1 and 2, let $\vec{S}^{(1)}=\frac{\hbar}{2}\vec{\sigma}^{(1)}$ and $\vec{S}^{(2)}=\frac{\hbar}{2}\vec{\sigma}^{(2)}$ denote the corresponding spin operators. Here $\vec{\sigma}\equiv\left(\sigma_x,\sigma_y,\sigma_z\right)$ and $\sigma_x,\sigma_y,\sigma_z$ are the three Pauli matrices.

[CSIR June 2011]

A. In the standard basis the matrices for the operators $S_x^{(1)}S_y^{(2)}$ and $S_y^{(1)}S_x^{(2)}$ respectively,

(a) $\frac{\hbar^2}{4}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix},\frac{\hbar^2}{4}\begin{pmatrix}-1 & 0\\ 0 & 1\end{pmatrix}$

(b) $\frac{\hbar^2}{4}\begin{pmatrix}i & 0\\ 0 & -i\end{pmatrix},\frac{\hbar^2}{4}\begin{pmatrix}-i & 0\\ 0 & i\end{pmatrix}$

(c) $\frac{\hbar^2}{4}\begin{pmatrix}0 & 0 & 0 & -i\\ 0 & 0 & i & 0\\ 0 & -i & 0 & 0\\ i & 0 & 0 & 0\end{pmatrix},\frac{\hbar^2}{4}\begin{pmatrix}0 & 0 & 0 & -i\\ 0 & 0 & -i & 0\\ 0 & i & 0 & 0\\ i & 0 & 0 & 0\end{pmatrix}$

(d) $\frac{\hbar^2}{4}\begin{pmatrix}0 & 1 & 0 & 0\\ 1 & 0 & 0 & 0\\ 0 & 0 & 0 & -i\\ 0 & 0 & i & 0\end{pmatrix},\frac{\hbar^2}{4}\begin{pmatrix}0 & -i & 0 & 0\\ i & 0 & 0 & 0\\ 0 & 0 & 0 & 1\\ 0 & 0 & 1 & 0\end{pmatrix}$

B. These two operators satisfy the relation

(a) $\left\{S_x^{(1)}S_y^{(2)},S_y^{(1)}S_x^{(2)}\right\}=S_z^{(1)}S_z^{(2)}$

(b) $\left\{S_x^{(1)}S_y^{(2)},S_y^{(1)}S_x^{(2)}\right\}=0$

(c) $\left[S_x^{(1)}S_y^{(2)},S_y^{(1)}S_x^{(2)}\right]=i\,S_z^{(1)}S_z^{(2)}$

(d) $\left[S_x^{(1)}S_y^{(2)},S_y^{(1)}S_x^{(2)}\right]=0$

2. The minimum energy of a collection of 6 non-interacting electrons of spin $-\frac{1}{2}$ placed in a one dimensional infinite square well potential of width L is **[CSIR Dec 2012]**

(a) $14\pi^2\hbar^2/mL^2$ (b) $91\pi^2\hbar^2/mL^2$ (c) $7\pi^2\hbar^2/mL^2$ (d) $3\pi^2\hbar^2/mL^2$

3. Consider a system of two non-interacting identical fermions, each of mass *m* in an infinite square well potential of width *a*. (Take the potential inside the well to be zero and ignore spin). The composite wavefunction for the system with total energy

$$E=\frac{5\pi^2\hbar^2}{2ma^2}$$

is **[CSIR June 2014]**

(a) $\frac{2}{a}\left[\sin\left(\frac{\pi x_1}{a}\right)\sin\left(\frac{2\pi x_2}{a}\right)-\sin\left(\frac{2\pi x_1}{a}\right)\sin\left(\frac{\pi x_2}{a}\right)\right]$

(b) $\frac{2}{a}\left[\sin\left(\frac{\pi x_1}{a}\right)\sin\left(\frac{2\pi x_2}{a}\right)+\sin\left(\frac{2\pi x_1}{a}\right)\sin\left(\frac{\pi x_2}{a}\right)\right]$

(c) $\frac{2}{a}\left[\sin\left(\frac{\pi x_1}{a}\right)\sin\left(\frac{3\pi x_2}{2a}\right)-\sin\left(\frac{3\pi x_1}{2a}\right)\sin\left(\frac{\pi x_2}{a}\right)\right]$

(d) $\frac{2}{a}\left[\sin\left(\frac{\pi x_1}{a}\right)\cos\left(\frac{\pi x_2}{a}\right)-\sin\left(\frac{\pi x_2}{a}\right)\sin\left(\frac{\pi x_2}{a}\right)\right]$

4. Three identical spin-$\frac{1}{2}$ particles of mass m are confined to a one-dimensional box of length L, but are otherwise free. Assuming that they are non-interacting, the energies of the lowest two energy eigenstates, in units of $\frac{\pi^2\hbar^2}{2mL^2}$, are **[CSIR Dec. 2018]**

(a) 3 and 6 (b) 6 and 9 (c) 6 and 11 (d) 3 and 9

5. The Hamiltonian of two interacting particles, one with spin-1 and the other with spin-$\frac{1}{2}$, is given by

$$H = A\,S_1 \cdot S_2 + B(S_{1x} + S_{2x}),$$

where S_1 and S_2 denote the spin operators of the first and second particles, respectively, and A and B are positive constants. The largest eigenvalue of this Hamiltonian is **[CSIR Dec. 2019]**

(a) $\frac{1}{2}(A\hbar^2 + 3B\hbar)$ (b) $3A\hbar^2 + B\hbar$ (c) $\frac{1}{2}(3A\hbar^2 + B\hbar)$ (d) $A\hbar^2 + 3B\hbar$

6. Two spin-$\frac{1}{2}$ fermions of mass m are confined to move in a one-dimensional infinite potential well of width L. If the particles are known to be in a spin triplet state, the ground state energy of the system (in units of $\frac{\hbar^2\pi^2}{2mL^2}$) is

(a) 8 (b) 2 (c) 3 (d) 5 **[CSIR Dec. 2019]**

7. The coordinates of two identical particles of spin 0 in one dimension are x_1 and x_2, respectively. Which of the following functions is an admissibe wavefunction fo the two-particle system? **[CSIR Dec. 2019]**

(a) $\frac{1}{2\pi\alpha} e^{-\frac{1}{2\alpha^2}\left(x_1^2+\frac{x_2^2}{4}\right)}$ (b) $\frac{1}{\pi\alpha} e^{i(k_1x_1+k_2x_2)} e^{-\frac{1}{2\alpha^2}\left(x_1+x_2^2\right)}$

(c) $\frac{1}{2\pi\alpha} e^{i(k_1x_1+k_2x_2)} e^{-\frac{2}{2\alpha^2}\left(x_1^2+\frac{x_2^2}{4}\right)}$ (d) $\frac{1}{21\pi\alpha^5}\left(x_1-x_2\right)^2 e^{-\frac{1}{2\alpha^2}\left(x_1^2+x_2^2\right)}$

8. A one-dimensional box contains one spin 0 particle and four spin-$\frac{1}{2}$ particles. All particles have the same mass, hence the single particle energies are $E_n = n^2E$, $n = 1, 2, 3, \ldots$ The energies of the ground and the first excited states of the system of particle energies of the system of particles are, respectively, **[CSIR Dec. 2019]**

(a) 10 E and 12 E (b) 11 E and 20 E (c) 11 E and 14 E (d) 10 E and 13 E

ANSWER KEY

1_A(c) **1_B(d)** **2. (a)** **3. (a)** **4. (b)** **5. (a)** **6. (d)**

7. (d) **8. (c)**

JEST Previous Years' Questions

1. The spatial part of a two-electron state is symmetric under exchange . If $|\uparrow\rangle$ and $|\downarrow\rangle$ represent the spin-up and spin-down states respectively of each particle, the spin-part of the two-particle state is **[JEST 2012]**

(a) $|\uparrow\rangle|\uparrow\rangle$ (b) $|\uparrow\rangle|\downarrow\rangle$

(c) $\left(|\downarrow\rangle|\uparrow\rangle-|\uparrow\rangle|\downarrow\rangle\right)/\sqrt{2}$ (d) $\left(|\downarrow\rangle|\uparrow\rangle+|\uparrow\rangle|\downarrow\rangle\right)/\sqrt{2}$

2. Consider a system of two spin-1/2 particles with total spin $\vec{S}=\vec{S}_1+\vec{S}_2$, where S_1 and S_2 are in terms of Pauli matrices σ_i. The spin triplet projection operator is **[JEST 2012]**

(a) $\frac{1}{4}+s_1.s_2$ (b) $\frac{3}{4}-s_1.s_2$ (c) $\frac{3}{4}+s_1.s_2$ (d) $\frac{1}{4}-s_1.s_2$

3. The grond state energy of 5 identical spin-1/2 particles which are subject to a one-dimensional simple harmonic oscillator potential of frequency ω is **[JEST 2012]**

(a) $\frac{15}{2}\hbar\omega$ (b) $\frac{13}{2}\hbar\omega$ (c) $\frac{1}{2}\hbar\omega$ (d) $5\hbar\omega$

4. Two electrons are confined in a one dimensional box of length L. The one-electron states are given by $\psi_n(x)=\sqrt{2/L}\sin(n\pi x/L)$. What would be the ground state wave function $\psi(x_1,x_2)$ if both electrons are arranged to have the same spin state? **[JEST 2013]**

(a) $\psi(x_1,x_2)=\frac{1}{\sqrt{2}}\left[\frac{2}{L}\sin\left(\frac{\pi x_1}{L}\right)\sin\left(\frac{2\pi x_2}{L}\right)+\frac{2}{L}\sin\left(\frac{2\pi x_1}{L}\right)\sin\left(\frac{\pi x_2}{L}\right)\right]$

(b) $\psi(x_1,x_2)=\frac{1}{\sqrt{2}}\left[\frac{2}{L}\sin\left(\frac{\pi x_1}{L}\right)\sin\left(\frac{2\pi x_2}{L}\right)-\frac{2}{L}\sin\left(\frac{2\pi x_1}{L}\right)\sin\left(\frac{\pi . x_2}{L}\right)\right]$

(c) $\psi(x_1,x_2)=\frac{2}{L}\sin\left(\frac{\pi x_1}{L}\right)\sin\left(\frac{2\pi x_2}{L}\right)$

(d) $\psi(x_1,x_2)=\frac{2}{L}\sin\left(\frac{2\pi x_1}{L}\right)\sin\left(\frac{\pi x_2}{L}\right)$

5. A simple model of a helium-like atom with electron-electron interaction is replaced by Hooke's law force is described by hamiltonian $\frac{-\hbar^2}{2m}\left(\nabla_1^2+\nabla_2^2\right)+\frac{1}{2}m\omega^2\left(r_1^2+r_2^2\right)-\frac{\lambda}{4}m\omega^2\left|\vec{r}_1-\vec{r}_2\right|^2$. What is the exact ground state energy? **[JEST 2013]**

(a) $E=\frac{3}{2}\hbar\omega\left(1+\sqrt{1+\lambda}\right)$ (b) $E=\frac{3}{2}\hbar\omega\left(1+\sqrt{\lambda}\right)$

(c) $E=\frac{3}{2}\hbar\omega\sqrt{1-\lambda}$ (d) $E=\frac{3}{2}\hbar\omega\left(1+\sqrt{1-\lambda}\right)$

6. The lowest quantum mechanical energy of a particle confined in a one-dimensional box of size L is 2 eV. The energy of the quantum mechanical ground state for a system of three non-interacting spin-1/2 particles is **[JEST 2014]**

(a) 6 eV (b) 10 eV (c) 12 eV (d) 16 eV

7. Suppose the spin degrees of freedom of a 2-particle system can be described by a 21-dimensional Hilbert subspace. Which among the following could be the spin of one of the particles ? **[JEST 2017]**

(a) $\frac{1}{2}$ (b) 3 (c) $\frac{3}{2}$ (d) 2

8. What is the difference between the maximum and the minimum eigenvalues of a system of two electrons whose Hamiltonian is $H + J\vec{S}_1 \cdot \vec{S}_2$, where $\vec{S}_1$ and $\vec{S}_2$ are the corresponding spin angular momentum operators of the two electrons? **[JEST 2018]**

(a) $J/4$ (b) $J/2$ (c) $3J/4$ (d) J

9. Suppose the spin degree of freedom of two particles (non-zero rest and mass and non-zero spin) is described completely by a Hilbert space of dimension twenty one. Which of the following could be the spin of one of the particles? **[JEST 2018]**

(a) 2 (b) 3/2 (c) 1 (d) 1/2

10. Consider two coupled harmonic oscillators of mass m each. The Hamiltonian describing the oscillators is

$$\hat{H} = \frac{\hat{p}_1^2}{2m} + \frac{\hat{p}_2^2}{2m} + \frac{1}{2}m\omega^2\left(\hat{x}_1^2 + \hat{x}_2^2 + (\hat{x}_1 - \hat{x}_2)^2\right)$$

The eigenvalues of $\hat{H}$ are given by (with n_1 and n_2 being non-negative integers)

(a) $E_{n_1,n_2} = \hbar\omega(n_1 + n_2 + 1)$ (b) $E_{n_1,n_2} = \hbar\omega\left(n_1 + \frac{1}{2}\right) + \frac{1}{\sqrt{3}}\hbar\omega\left(n_2 + \frac{1}{2}\right)$

(c) $E_{n_1,n_2} = \hbar\omega\left(n_1 + \frac{1}{2}\right) + \sqrt{3}\hbar\omega\left(n_2 + \frac{1}{2}\right)$ (d) $E_{n_1,n_2} = \frac{1}{\sqrt{3}}\hbar\omega(n_1 + n_2 + 1)$ **[JEST 2018]**

11. Consider a system of 15 non-interacting spin-polarized electrons. They are trapped in a two dimensional isotropic harmonic oscillator potential $V(x, y) = \frac{1}{2}m\omega^2(x^2 + y^2)$. The angular frequency ω is such that $\hbar\omega = 1$ in some chosen unit. What is the ground state energy of the system in the same units ? **[JEST 2019]**

ANSWER KEY

1. (c) **2. (c)** **3. (b)** **4. (b)** **5. (d)** **6.(c)** **7. (b)**

8. (d) **9. (c)** **10. (c)** **11. (55)**

TIFR Previous Years' Questions

1. Two identical non-interacting particles, each of mass m and spin 1/2, are placed in a one-dimensional box of length L. In quantum mechanics, the lowest possible value of the total energy of these two particles is ε_0. If, instead, four such particles are introduced into a similar one-dimensional box of length 2L, then the lowest possible value of their total energy will be **[TIFR 2011]**

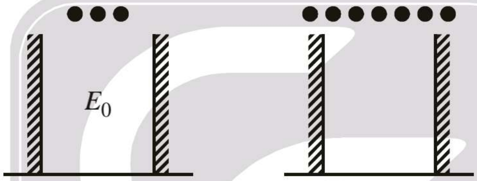
   (a) $2\varepsilon_0$ (b) $5\varepsilon_0/4$ (c) $3\varepsilon_0/2$ (d) ε_0

2. Consider two spin-1/2 identical particles A and B, separated by a distance r, interacting through a potential $V(r)=\frac{V_0}{r}\vec{S}_A.\vec{S}_B$ where V_0 is a positive constant and the spins are $\vec{S}_{A,B}=\vec{\sigma}=(\sigma_x,\sigma_y,\sigma_z)$ in terms of the Pauli spin matrices. The exception values of this potential in the spin-singlet and triplet states are

   (a) $\text{Singlet}:-\frac{V_0}{3r},\ \text{Triplet}:\frac{V_0}{r}$ (b) $\text{Singlet}:-\frac{3V_0}{r},\ \text{Triplet}:\frac{V_0}{r}$ **[TIFR 2016]**

   (c) $\text{Singlet}:\frac{3V_0}{r},\ \text{Triplet}:-\frac{V_0}{r}$ (d) $\text{Singlet}:-\frac{V_0}{r},\ \text{Triplet}:\frac{3V_0}{r}$

3. A quantum mechanical system consists of one-dimensional infinite box, as indicated in the figure below.



   3 (three) identical non-interacting spin-$\frac{1}{2}$ particles, are first placed in the box, and the ground state energy of the system is found to be $E_0=18\text{ eV}$. If 7 (seven) such identical particles are placed in the box, what will be the ground state energy, in units of eV ? **[TIFR 2017]**

4. A system of two spin-$\frac{1}{2}$ particles 1 and 2 has the Hamiltonian $\hat{H}=\varepsilon_0\,\hat{h}_1\otimes\hat{h}_2$, where

$$\hat{h}_1=\begin{pmatrix}2 & 0\\ 0 & 1\end{pmatrix},\ \hat{h}_2=\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}$$

   and ε_0 is a constant with the dimension of energy. The ground state of this system has energy

   (a) $\sqrt{2}\,\varepsilon_0$ (b) $-4\varepsilon_0$ (c) 0 (d) $-2\varepsilon_0$ **[TIFR 2019]**

ANSWER KEY

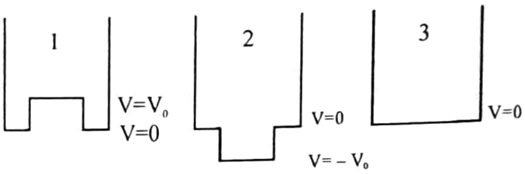
1. (b) **2. (b)** **3. (132)** **4. (d)**

GATE Previous Years' Questions

~~1~~ 363 Let E'_n $(n = 0, 1, 2, ...)$ be the energy eigenvalues for a particle of mass m placed in an anharmonic potential $V(x) = \frac{1}{2}m\omega^2x^2 + ax^4, (a > 0)$. Let $E_n \equiv \left(n + \frac{1}{2}\right)\hbar\omega$. Then according to the first order perturbation theory: **[GATE 1996]**

(a) $E'_0 = E_0$ (b) $E'_0 > E_0$ (c) $E'_0 < E_0$ (d) $E'_0 < E_0$ for all n

~~2~~ 365 Let E_1, E_2, E_3 be the respective ground state energies of the following potentials: Which one of the following is correct? **[GATE 2000]**



(a) $E_1 < E_2 < E_3$ (b) $E_3 < E_1 < E_2$ (c) $E_2 < E_3 < E_1$ (d) $E_2 < E_1 < E_3$.

~~3~~ 363 The wave function of a one-dimensional harmonic oscillator is $\psi_0 = A\exp\left(\frac{-\alpha^2x^2}{2}\right)$ for the ground state. For the purterbation $E_0(\alpha x/10)^4$, the first order change in the ground state energy is: [Given: $\left[\Gamma(x+1) = \int_0^\infty t^x \exp(-t)dt\right]$ **[GATE 2004]**

(a) $\left(\frac{1}{2}E_0\right)10^{-4}$ (b) $(3E_0)10^{-4}$ (c) $\left(\frac{3}{4}E_0\right)10^{-4}$ (d) $(E_0)10^{-4}$

371 **Common data for Q. 4, Q. 5 and Q.6**

An unperturbed two-level system has energy eigenvalues E_1 and E_2, and eigen functions $\begin{pmatrix}1\\0\end{pmatrix}$ and $\begin{pmatrix}0\\1\end{pmatrix}$.

When perturbed, its Hamiltonian is represented by $\begin{pmatrix}E_1 & A\\ A^* & E_2\end{pmatrix}$ **[GATE 2006]**

4. The first-order correction to E_1 is:
(a) 4A (b) 2A (c) A (d) 0

5. The second-order correction to E_1 is

(a) 0 (b) A (c) $\frac{A^2}{E_2 - E_1}$ (d) $\frac{|A|^2}{E_1 - E_2}$

✓6. The first-order correction to the eigenfunction $\begin{pmatrix}1\\0\end{pmatrix}$ is:

(a) $\begin{pmatrix}0\\ A^*/(E_1 - E_2)\end{pmatrix}$ (b) $\begin{pmatrix}0\\1\end{pmatrix}$ (c) $\begin{pmatrix}A^*/(E_1 - E_2)\\ 0\end{pmatrix}$ (d) $\begin{pmatrix}1\\1\end{pmatrix}$

~~7.~~ 365 A particle of mass m is confined in an infinite potential well $V(x)=\begin{cases}0 & \text{if } 0<x<L,\\ \infty & \text{otherwise}\end{cases}$

It is subjected to a perturbing potential $V_p(x)=V_0\sin\left(\frac{2\pi x}{L}\right)$ within the well. Let $E^{(1)}$ and $E^{(2)}$ be the corrections to the ground state energy in the first and second order in V_0, respectively. Which of the following are true? **[GATE 2010]**

(a) $E^{(1)}=0;\ E^{(2)}<0$ (b) $E^{(1)}>0;\ E^{(2)}=0$

(c) $E^{(1)}=0;\ E^{(2)}$ depends on the sign of V_0 (d) $E^{(1)}<0;\ E^{(2)}<0$

~~8.~~ 366 The normalized eigenstates of a particle in a one-dimensional potential well

$$V(x)=\begin{cases}0 & \text{if } 0<x<a\\ \infty & \text{otherwise}\end{cases}$$

are given by

$$\psi_n(x)=\sqrt{\frac{2}{a}}\sin\left(\frac{n\pi x}{a}\right)$$

The particle is subjected to a perturbation

$$V'(x)=V_0\cos\left(\frac{\pi x}{a}\right)\quad \text{for } 0\le x\le\frac{a}{2}$$

[GATE 2011]

$$=0 \qquad \text{otherwise}$$

The shift in the ground state energy due tothe perturbation, in the first order perturbation theory,

(a) $\frac{2V_0}{3\pi}$ (b) $\frac{V_0}{3\pi}$ (c) $-\frac{V_0}{3\pi}$ (d) $-\frac{2V_0}{3\pi}$

~~9.~~ 377 Consider a system in the unperturbed state described by the Hamiltonian, $H_0=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$. The system is subjected to a perturbation of the form $H'=\begin{pmatrix}\delta & \delta\\ \delta & \delta\end{pmatrix}$, where $\delta<<1$. The energy eigenvalues of the perturbed system using the first order perturbation approximation are **[GATE 2012]**

(a) 1 and $(1+2\delta)$ (b) $(1+\delta)$ and $(1-\delta)$ (c) $(1+2\delta)$ and $(1-2\delta)$ (d) $(1+\delta)$ and $(1-2\delta)$

371 **Common Data for Q. 10 and Q. 11:**

To the given unperturbed Hamiltonian $\begin{bmatrix}5 & 2 & 0\\ 2 & 5 & 0\\ 0 & 0 & 2\end{bmatrix}$

we add a small perturbation given by $\varepsilon\begin{bmatrix}1 & 1 & 1\\ 1 & 1 & -1\\ 1 & -1 & 1\end{bmatrix}$ **[GATE 2013]**

where ε is a small quantity.

~~10.~~ The ground state eigen vector of the unperturbed Hamiltonian is

(a) $(1/\sqrt{2},1/\sqrt{2},0)$ (b) $(1/\sqrt{2},-1/\sqrt{2},0)$ (c) (0, 0, 1) (d) (1, 0, 0)

~~11.~~ A pair of eigen values of the perturbed Hamiltonian, using first order perturbation theory, is

(a) $3+2\varepsilon, 7+2\varepsilon$ (b) $3+2\varepsilon, 2+\varepsilon$ (c) $3, 7+2\varepsilon$ (d) $3, 2+2\varepsilon$

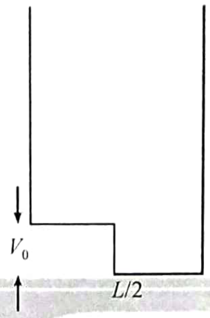
~~12.~~ 363 A particle is confined in a one-dimensional potential box with the potential

$$V(x)=\begin{cases}0 & \text{if } 0<x<a \\ \infty & \text{otherwise}\end{cases}$$

If the particle is subjected to a perturbation, within the box, $W=\beta x$, where β is a small constant, the first order correction to the ground state energy is **[GATE 2014]**

(a) 0 (b) $\frac{a\beta}{4}$ (c) $\frac{a\beta}{2}$ (d) $a\beta$

13. ✓ 367 89me 360 A particle is confined in a box of length L as shown below. **[GATE 2015]**



If the potential V_0 is treated as a perturbation, including the first order correction, the ground state energy is

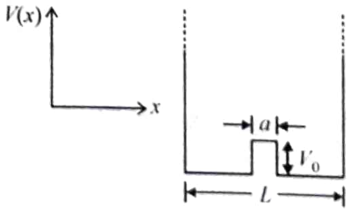
(a) $E=\frac{\hbar^2\pi^2}{2mL^2}+V_0$ (b) $E=\frac{\hbar^2\pi^2}{2mL^2}-\frac{V_0}{2}$ (c) $E=\frac{\hbar^2\pi^2}{2mL^2}+\frac{V_0}{4}$ (d) $E=\frac{\hbar^2\pi^2}{2mL^2}+\frac{V_0}{2}$

~~14.~~ 383 A hydrogen atom is in its ground state. In the presence of a uniform electric field $\vec{E}=E_0\hat{z}$, the leading order change in its energy is proportional to $(E_0)^n$. The value of the exponent n is _______. **[GATE 2016]**

~~15.~~ 363 A one dimensional simple harmonic oscillator with Hamiltonian $H_0=\frac{p^2}{2m}+\frac{1}{2}kx^2$ is subjected to a small perturbation, $H_1=\alpha x+\beta x^3+\gamma x^4$. The first order correction to the ground state energy is dependent on

(a) only β (b) α and γ (c) α and β (d) only γ **[GATE 2017]**

~~16.~~ 369 The ground state energy of a particle of mass m in an infinite potential well is E_0. It changes to $E_0(1+\alpha\times10^{-3})$, when there is a small potential bump of height $V_0=\frac{\pi^2\hbar^2}{50mL^2}$ and width $a=L/100$, as shown in the figure. The value of α is _______ (up to two decimal places). **[GATE 2018]**



Compare with prob.-378

page-379 से (matrix element देखो

~~17~~ 383 An electric field $\vec{E} = E_0\hat{z}$ is applied to a hydrogen atom in $n = 2$ excited state. Ignoring spin, the $n = 2$ state is fourfold degenerate, which in the $|\ell, m\rangle$ basis are given by $|0,0\rangle, |1,1\rangle, |1,0\rangle$ and $|1,-1\rangle$. The H' is the interaction Hamiltonian corresponding to the applied electric field, which of the following matrix elements is non-zero? **[GATE 2019]**

(a) $\langle 0,0|H'|0,0\rangle$ (b) $\langle 0,0|H'|1,1\rangle$ ✓(c) $\langle 0,0|H'|1,0\rangle$ (d) $\langle 0,0|H'|1,-1\rangle$

~~18~~ 375 The Hamiltonian of a system is $H = \begin{pmatrix} 1 & \varepsilon \\ \varepsilon & -1 \end{pmatrix}$ with $\varepsilon << 1$. The fourth order contribution to the ground state energy of H is $\gamma\varepsilon^4$. The value of γ (rounded off to three decimal places) is __________ **[GATE 2019]**

ANSWER KEY

1. (b) **2.(c)** **3. (c)** **4. (d)** **5. (d)** **6. (a)** **7. (a)**
8. (a) **9. (a)** **10. (c)** **11. (c)** **12. (c)** **13. (d)** **14. (2)**
15. (d) **16. (0.78 to 0.82)** **17. (c)** **18. (0.125)**

475
✓ Ques.19

CSIR-UGC-NET Previous Years' Questions

~~1.~~ 364 If the perturbation $H' = ax$, where a is a constant; is added to the infinite square well potential

$$V(x) = \begin{cases} 0 \text{ for } & 0 \le x \le \pi \\ \infty & \text{otherwise} \end{cases}$$

The first order correction to ground state energy is: **[CSIR June 2011]**

(a) $\frac{a\pi}{2}$ (b) $a\pi$ (c) $\frac{a\pi}{4}$ (d) $\frac{a\pi}{\sqrt{2}}$

~~2.~~ 364 The perturbation $H' = bx^4$, where 'b' is a constant, is added to the one dimensional harmonic oscillator potential $V(x) = \frac{1}{2}m\omega^2x^2$. Which of the following denotes the correction to the ground state energy to first order in b? **[CSIR Dec 2011]**

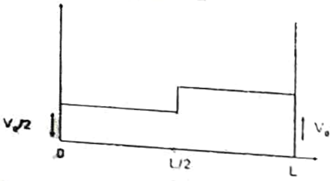
[Hint: The normalized ground state wave function of the one dimensional harmonic oscillator potential is $\psi_0 = \left(\frac{m\omega}{\hbar\pi}\right)^{1/4} e^{-m\omega x^2/2\hbar}$. You may use the following integral $\int_{-\infty}^{\infty} x^{2n} e^{-ax^2} dx = a^{-n-\frac{1}{2}}\Gamma\left(n+\frac{1}{2}\right)$

(a) $\frac{3b\hbar^2}{4m^2\omega^2}$ (b) $\frac{3b\hbar^2}{2m^2\omega^2}$ (c) $\frac{3b\hbar^2}{2\pi m^2\omega^2}$ (d) $\frac{b\hbar}{4m\omega}$

~~3.~~ 364 A constant perturbation as shown in the figure below acts on a particle of mass *m* confined in a infinite potential well between 0 and *L*. The first-order correction to the ground state energy of the particle is **[CSIR Dec 2011]**

(a) $\frac{V_0}{2}$ (b) $\frac{3V_0}{4}$

(c) $\frac{V_0}{4}$ (d) $\frac{3V_0}{2}$



~~4.~~ 374 Consider an electron in a box of lenght L with periodic boundary condition $\psi(x) = \psi(x+L)$. If the electron is in the $\psi_k(x) = \frac{1}{\sqrt{L}}e^{jkx}$ with energy $\varepsilon_k = \frac{\hbar^2k^2}{2m}$, what is the correction to its energy, to second order of perturbation theory, when it is subjected to weak periodic potential $V(x) = V_0 \cos gx$, where g is an integral multiple of the $2\pi/L$? **[CSIR June 2012]**

(a) $V_0^2\, \varepsilon_g/\varepsilon_k^2$ (b) $-\frac{mV_0^2}{2\hbar^2}\left(\frac{1}{g^2+2kg}+\frac{1}{g^2-2kg}\right)$

(c) $V_0^2\left(\varepsilon_k-\varepsilon_g\right)/\varepsilon_g^2$ (d) $V_0^2/\left(\varepsilon_k+\varepsilon_g\right)$

5. 362 The energy eigenvalues of a particle in the potential $V(x)=\frac{1}{2}m\omega^2x^2-ax$ are **[CSIR Dec 2012]**

(a) $E_n=\left(n+\frac{1}{2}\right)\hbar\omega-\frac{a^2}{2m\omega^2}$ (b) $E_n=\left(n+\frac{1}{2}\right)\hbar\omega+\frac{a^2}{2m\omega^2}$

(c) $E_n=\left(n+\frac{1}{2}\right)\hbar\omega-\frac{a^2}{m\omega^2}$ (d) $E_n=\left(n+\frac{1}{2}\right)\hbar\omega$

6. 364 The perturbation $H'=\begin{cases} b(a-x) & -a<x<a \\ 0 & \text{otherwise}\end{cases}$

acts on a particle of mass 'm' confined in an infinite square well potential

$$V(x)=\begin{cases} 0 & -a<x<a \\ \infty & \text{otherwise}\end{cases}$$

The first order correction to the ground state energy of the particle is **[CSIR Dec 2012]**

(a) $\frac{ba}{2}$ (b) $\frac{ba}{\sqrt{2}}$ (c) $2ba$ (d) ba

7. 383 Consider a two-dimensional infinite square will $V(x,y)=\begin{cases} 0, & \text{if } 0<x<a,\ \ 0<y<a \\ \infty, & \text{otherwise}\end{cases}$. Its normalized eigenfunctions are $\psi_{n_x,n_y}(x,y)=\frac{2}{a}\sin\left(\frac{n_x\pi x}{a}\right)\sin\left(\frac{n_y\pi y}{a}\right)$ where $n_x, n_y = 1, 2, 3, \ldots$.. If a perturbation $H'=\begin{cases} V_0, & \text{if } 0<x<\frac{a}{2},\ \ 0<y<\frac{a}{2} \\ 0, & \text{otherwise}\end{cases}$ is applied, then the correction to the energy of the first excited state to order V_0 is **[CSIR June 2013]**

(a) $\frac{V_0}{4}$ (b) $\frac{V_0}{4}\left[1\pm\frac{64}{9\pi^2}\right]$ (c) $\frac{V_0}{4}\left[1\pm\frac{16}{9\pi^2}\right]$ (d) $\frac{V_0}{4}\left[1\pm\frac{32}{9\pi^2}\right]$

8. 439 Two identical bosons of mass *m* are placed in a one-dimensional potential $V(x)=\frac{1}{2}m\omega^2x^2$. The bosons interact via a weak potential, $V_{12}=V_0\exp[-m\Omega(x_1-x_2)^2/4\hbar]$ where x_1 and x_2 denote coordinates of the particles. Given that the ground state wavefunction of the harmonic oscillator is $\psi_0(x)=\left(\frac{m\omega}{\pi\hbar}\right)^{\frac{1}{4}}e^{-\frac{m\omega x^2}{2\hbar}}$.

The ground state energy of the two-boson system, to the first order in V_0, is: **[CSIR June 2013]**

(a) $\hbar\omega+2V_0$ (b) $\hbar\omega+\frac{V_0\Omega}{\omega}$

(c) $\hbar\omega+V_0\left(1+\frac{\Omega}{2\omega}\right)^{-1/2}$ (d) $\hbar\omega+V_0\left(1+\frac{\omega}{\Omega}\right)$

~~9.~~ 365 The motion of a particle of mass m in one dimension is described by the Hamiltonian $H = \frac{p^2}{2m} + \frac{1}{2} m\omega^2 x^2 + \lambda x$. What is the difference between the (quantized) energies of the first two levels? (In the following, $\langle x \rangle$ is the expectation value of x in the ground state)

[CSIR Dec 2013]

(a) $\hbar\omega - \lambda\langle x \rangle$ (b) $\hbar\omega + \lambda\langle x \rangle$ (c) $\hbar\omega + \frac{\lambda^2}{2m\omega^2}$ (d) $\hbar\omega$

~~10.~~ 365 The ground state eigenfunction for the potential $V(x) = -\delta(x)$, where $\delta(x)$ is the delta function, is given by $\psi(x) = Ae^{-\alpha|x|}$, where A and $\alpha > 0$ are constants. If a perturbation $H' = bx^2$ is applied, the first order correction to the energy of the ground state will be **[CSIR June 2014]**

(a) $\frac{b}{\sqrt{2}\alpha^2}$ (b) $\frac{b}{\alpha^2}$ (c) $\frac{2b}{\alpha^2}$ (d) $\frac{b}{2\alpha^2}$

~~11.~~ 377 The Hamiltonian H_0 for a three-state quantum system is given by the matrix $H_0 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}$. When perturbed by $H' = \in \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$ where $\in \ll 1$, the resulting shift in the energy eigenvalue $E_0 = 2$ is

[CSIR Dec 2014]

(a) $\in, -2\in$ (b) $-\in, 2\in$ (c) $\pm\in$ (d) $\pm 2\in$

~~12.~~ 369 A hydrogen atom is subjected to the perturbation $V_{pert}(r) = \in \cos 2r / a_0$ where a_0 is the Bohr radius. The change in the ground state energy to first order in $\in$ is **[CSIR Dec. 2015]**

(a) $\in/4$ (b) $\in/2$ (c) $-\in/2$ (d) $-\in/4$

~~13.~~ 368 Consider a particle of mass m in a potential $V(x) = \frac{1}{2} m\omega^2 x^2 + g\cos kx$. The change in the ground state energy, compared to the simple harmonic potential $\frac{1}{2} m\omega^2 x^2$, to first order in g is **[CSIR June 2016]**

(a) $g\exp\left(-\frac{k^2\hbar}{2m\omega}\right)$ (b) $g\exp\left(\frac{k^2\hbar}{2m\omega}\right)$ (c) $g\exp\left(-\frac{2k^2\hbar}{m\omega}\right)$ (d) $g\exp\left(-\frac{k^2\hbar}{4m\omega}\right)$

14. 4.35 A particle of charge q in one dimension is in a simple harmonic potential with angular frequency ω. It is subjected to a time dependent electric field $E(t) = Ae^{-(t/\tau)^2}$, where A and τ are positive constants and $\omega\tau \gg 1$. If in the distant past $t \to -\infty$ the particle was in its ground state, the probability that it will be in the first excited state as $t \to +\infty$ is proportional to **[CSIR Dec. 2016]**

(a) $e^{-\frac{1}{2}(\omega\tau)^2}$ (b) $e^{\frac{1}{2}(\omega\tau)^2}$ (c) 0 (d) $\frac{1}{(\omega\tau)^2}$

15. 435 A constant perturbation H' is applied to a system for time Δt (where $H'\Delta t \ll h$) leading to a transition from a state with energy E_i to another with energy E_f. If the time of application is doubled, the probability of transition will be **[CSIR June 2017]**

(a) unchanged (b) doubled (c) quadrupled (d) halved

16. 368 The Coulomb potential $V(r) = -e^2/r$ of a hydrogen atom is perturbed by adding $H' = bx^2$ (where b is a constant) to the Hamiltonian. The first order correction to the ground state energy is (The ground state wavefunction is $\psi_0 = \frac{1}{\sqrt{\pi a_0^3}} e^{-r/a_0}$). **[CSIR June 2017]**

(a) $2ba_0^2$ (b) ba_0^2 (c) $ba_0^2/2$ (d) $\sqrt{2}ba_0^2$

17. 369 Consider a one-dimensional infinite square well **[CSIR Dec. 2017]**

$$V(x) = \begin{cases} 0 & \text{for} \quad 0 < x < a, \\ \infty & \quad\quad \text{otherwise} \end{cases}$$

If a perturbation

$$\Delta V(x) = \begin{cases} V_0 & \text{for} \quad 0 < x < \frac{a}{3}, \\ 0 & \quad\quad \text{otherwise} \end{cases}$$

is applied, then the correction to the energy of the first excited state, to first order in ΔV, is nearest to

(a) V_0 (b) $0.16\ V_0$ (c) $0.2\ V_0$ [✓ (d) $0.33\ V_0$]

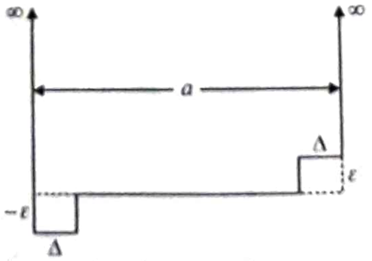
18. 384 A particle of mass m is constrained to move in a circular ring to radius R. When a perturbation

$$V' = \frac{a}{R^2}\cos^2\phi$$

(where a is a real constant) is added, the shift in energy of the ground state, to first order in a, is

(a) a/R^2 (b) $2a/R^2$ (c) $a/(2R^2)$ (d) $a/(\pi R^2)$ **[CSIR June 2018]**

19. 369 The infinite square-well potential of a particle in a box of size a is modified as shown in the figure below (assume $\Delta \ll a$) **[CSIR June 2019]**



The energy of the ground state, compared to the ground state energy before the perturbation was added

(a) increases by a term of order ε (b) decreases by a term of order ε

(c) increases by a term of order ε^2 (d) decreases by a term of order ε^2

476✓

~~20.~~ 440✓ A charged, spin-less particle of mass m is subjected to an attractive potential **[CSIR June 2019]**

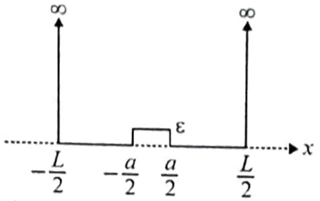
$V(x,y,z)=\frac{1}{2}k(x^2+y^2+z^2)$, where k is a positive constant.

Now a perturbation in the form of a weak magnetic field $\mathrm{B}=B_0\hat{k}$ (where B_0 is a constant) is switched on. Into how many distinct levels will the second excited state of the unperturbed Hamiltonian split?

(a) 5 (b) 4 (c) 2 (d) 1

870

~~21.~~ HW A quantum particle in a one-dimensional box of length L is perturbed by a barrier of height ε and length a, as shown in the figure. **[CSIR Dec. 2019]**



Assuming that ε is much smaller than the ground state energy of the particle in the unperturbed box and $a \ll L$. The changes of the two lowest energy eigenvalues, to first order in ε and first order in a, are, respectively.

(a) $\frac{2\varepsilon a}{L}$ and $-\frac{2\varepsilon a}{L}$ (b) 0 and $\frac{2\varepsilon a}{L}$ (c) $-\frac{2\varepsilon a}{L}$ and 0 (d) $\frac{2\varepsilon a}{L}$ and 0

ANSWER KEY

1. (a)	**2. (a)**	**3. (b)**	**4. (b)**	**5. (a)**	**6. (d)**	**7. (b)**
8. (c)	**9. (d)**	**10. (d)**	**11. (c)**	**12. (d)**	**13. (d)**	**14. (a)**
15. (b)	**16. (b)**	**17. (*)**	**18. (c)**	**19. (d)**	**20. (a)**	**21. (d)**

JEST Previous Years' Questions

~~1.~~ 373 Consider a spin-$\frac{1}{2}$ particle characterized by the Hamiltonian $H = \omega S_z$. Under a perturbation $H' = gS_x$, the second order correction to the ground state energy is given by, **[JEST 2015]**

(a) $-\frac{g^2}{4\omega}$ (b) $\frac{g^2}{4\omega}$ (c) $-\frac{g^2}{2\omega}$ (d) $\frac{g^2}{2\omega}$

~~2.~~ 364 A particle of mass m is confined in a potential well given by $V(x) = 0$ for $-L/2 < x < L/2$ and $V(x) = \infty$ elsewhere. A perturbing potential $H'(x) = \alpha x$ has been applied to the system. Let the first and second order corrections to the ground state be $E_0^{(1)}$ and $E_0^{(2)}$, respectively. Which one of the following statements is correct? **[JEST 2015]**

(a) $E_0^{(1)} < 0$ and $E_0^{(2)} > 0$ (b) $E_0^{(1)} = 0$ and $E_0^{(2)} > 0$

(c) $E_0^{(1)} > 0$ and $E_0^{(2)} < 0$ (d) $E_0^{(1)} = 0$ and $E_0^{(2)} < 0$

H.W ~~3.~~ 370 Consider a quantum particle of mass m in one dimension in an infinite potential well, i.e., $V(x) = 0$ for $-a/2 < x < a/2$, and $V(x) = \infty$ for $|x| \geq a/2$. A small perturbation, $V'(x) = 2\epsilon|x|/a$, is added. The change in the ground state energy to $O(\varepsilon)$ is: **[JEST 2016]**

(a) $\frac{\varepsilon}{2\pi^2}(\pi^2 - 4)$ (b) $\frac{\varepsilon}{2\pi^2}(\pi^2 + 4)$ (c) $\frac{\varepsilon\pi^2}{2}(\pi^2 + 4)$ (d) $\frac{\varepsilon\pi^2}{2}(\pi^2 - 4)$

~~4.~~ 366 A particle is described by the following Hamiltonian $\hat{H} = \frac{\hat{p}^2}{2m} + \frac{1}{2}m\omega^2\hat{x}^2 + \lambda\hat{x}^4$, where the quartic term can be treated perturbatively. If ΔE_0 and ΔE_1 denote the energy correction of $O(\lambda)$ to the ground state and the first excited state respectively, what is the fraction $\Delta E_1/\Delta E_0$? **[JEST 2017]**

~~5.~~ 370 A harmonic oscillator has the following Hamiltonian

$$H_0 = \frac{\hat{p}^2}{2m} + \frac{1}{2}m\omega^2\hat{x}^2$$

It is perturbed with a potential $V = \lambda\hat{x}^4$. Some of the matrix elements of $\hat{x}^2$ in terms of its expectation value in the ground state are given as follows:

$$\langle 0|\hat{x}^2|0\rangle = C;\ \langle 0|\hat{x}^2|2\rangle = \sqrt{2}C;\ \langle 1|\hat{x}^2|1\rangle = 3C;\ \langle 1|\hat{x}^2|3\rangle = \sqrt{6}C$$

where $|n\rangle$ is the normalized eigenstate of H_0 corresponding to the eigenvalue $E_n = \hbar\omega(n+1/2)$. Suppose ΔE_0 and ΔE_1 denote the energy corretion of $O(\lambda)$ to the ground state and the first excited state, respectively. What is the fraction $\Delta E_1/\Delta E_0$? **[JEST 2018]**

439

6. H.W. Consider a quantum particle of mass m and charge e moving in a two dimensional potential given as :

$$V(x,y)=\frac{k}{2}(x-y)^2+k(\boldsymbol{x}+y)^2$$

The particle is also subject to an external electric field $\vec{E}=\boldsymbol{\lambda}(\hat{i}-\hat{j})$, where λ is a constant. $\hat{i}$ and $\hat{j}$ corresponds to unit vectors along x and y directions, respectively. Let E_1 and E_0 be the energies of the first excited state and ground state, respectively. What is the value of E_1-E_0 ? **[JEST 2019]**

(a) $\hbar\sqrt{\frac{2k}{m}}$ (b) $\hbar\sqrt{\frac{2k}{m}}+e\lambda^2$ (c) $3\hbar\sqrt{\frac{2k}{m}}$ (d) $3\hbar\sqrt{\frac{2k}{m}}+e\lambda^2$

ANSWER KEY

1. (a) **2. (d)** **3. (a)** **4. (0005)** **5. (0005)** **6. (a)**

TIFR Previous Years' Questions

~~1.~~ 435 A charged particle is in the ground state of a one-dimensional harmonic oscillator potential, generated by electrical means. If the power is suddenly switched off, so that the potential disappears, then, according to quantum mechanics, **[TIFR 2010]**

(a) the particle will shoot out of the well and move out towards infinity in one of the two possible direction as

(b) the particle will stop oscillating and as time increases it may be found farther and farther way from the centre of the well

(c) the particle will keep oscillating about the same mean position but with increasing amplitude as time increases

(d) the particle will undergo a transition to one of the higher excited states of the harmonic oscillator

~~2.~~ 436 A particle of mass 'm' and charge 'e' is in the ground state of a one-dimensional harmonic oscillator potential in the presence of a uniform external electric field E. The total potential felt by the particle is

$$V(x)=\frac{1}{2}kx^2-eEx$$ **[TIFR 2014]**

If the electric field is suddenly switched off, then the particle will

(a) Make a transition to any harmonic oscillator state with $x=-eE/k$ as origin without emitting any photon.

(b) Make a transition to any harmonic oscillator state with $x=0$ as origin and absorb a photon.

(c) Settle into the harmonic oscillator ground state with $x=0$ as origin after absorbing a photon.

(d) Oscillate back and forth with initial amplitude eE/k, emitting multiple photons as it does so.

~~3.~~ 373 A quantum mechanical system which has stationary states $|1\rangle, |2\rangle$ and $|3\rangle$, corresponding to energy levels 0 eV, 1 eV and 2 eV respectively, is perturbed by a potential of the form $\hat{V}=\varepsilon|1\rangle\langle 3|+\varepsilon|3\rangle\langle 1|$, where, in eV, $0<\varepsilon\ll 1$.

The new ground state, correct to order ε, is approximately. **[TIFR 2017]**

(a) $\left(1-\frac{\varepsilon}{2}\right)|1\rangle+\frac{\varepsilon}{2}|3\rangle$ (b) $|1\rangle+\frac{\varepsilon}{2}|2\rangle-\varepsilon|3\rangle$ (c) $|1\rangle+\frac{\varepsilon}{2}|3\rangle$ (d) $|1\rangle-\frac{\varepsilon}{2}|3\rangle$

~~4.~~ 438 A particle of mass m moves in a two-dimensional space (x, y) under the influence of a Hamiltonian

$$H=\frac{1}{2m}\left(p_x^2+p_y^2\right)+\frac{1}{4}m\omega^2(5x^2+5y^2+6xy)$$

Find the ground state energy of this particle in a quantum-mechanical treatment. **[TIFR 2018]**

ANSWER KEY

1. (a) **2. (b)** **3. (d)** **4.** $\left(\frac{3}{2}h\omega\right)$

GATE Previous Years' Questions

1. A particle is scatterd by a central potential. If the dominant contribution to the scattering is from the p-wave, the differential cross-section is **[GATE 2000]**

(a) isotropic (b) proportional to $\cos^2\theta$
(c) stabilize the magnetization (d) cause critical magnetic fluctuations

2. If σ is the total cross-section and $f(\theta)$ is the scattering amplitude (θ being the angle of scattering) for a quantum mechanical elastic scattering by a spherically symmetric potential, then which of the following is true? Note that k is the magnitude of the wave vector along the $\hat{z}$ direction. **[GATE 2002]**

(a) $\sigma = |f(\theta)|^2$ (b) $\sigma = \frac{4\pi}{k}|f(\theta=0)|^2$

(c) $\sigma = \frac{4\pi}{k} \times$ imaginary part of $[f(\theta=0)]$ (d) $\sigma = \frac{4\pi}{k}|f(\theta)|^2$

Common data for Q. 3 and Q. 4.

A particle is scattered by a spherically symmetric potential. In the centre of mass (CM) frame the wavefunction of the incoming particle is $\psi = Ae^{ikz}$ where k is the wavevector and A is a constant. **[GATE 2005]**

3. If $f(\theta)$ is an angular function then in the asymptotic region the scattered wavefunction has the form

(a) $\frac{Af(\theta)e^{ikr}}{r}$ (b) $\frac{Af(\theta)e^{-ikr}}{r}$ (c) $\frac{Af(\theta)e^{ikr}}{r^2}$ (d) $\frac{Af(\theta)e^{-ikr}}{r^2}$

4. The differential scattering cross section $\sigma(\theta)$ in CM frame is:

(a) $\sigma(\theta) = |A|^2\frac{|f(\theta)|^2}{r^2}$ (b) $\sigma(\theta) = |A|^2|f(\theta)|^2$

(c) $\sigma(\theta) = |f(\theta)|^2$ (d) $\sigma(\theta) = |A||f(\theta)|$

5. The scattering of particles by a potential can be analyzed by Born approximation. In particular, if the scattered wave is replaced by an appropriate plane wave, the corresopnding Born approximation is known as the first Born approximation. Such as approximation is valid for **[GATE 2016]**

(a) large incident energies and weak scattering potentials
(b) large incident energies and strong scattering potentials.
(c) small incident energies and weak scattering potentials
(d) small incident energies and strong scattering potentials.

6. Consider an elastic scattering of particles in $l = 0$ states. If the corresponding phase shift δ_0 is 90° and the magnitude of the incident wave vector is equal to $\sqrt{2\pi}$ fm^{-1} then the total scattering cross section in units of fm^2 is ________________ **[GATE 2016]**

ANSWER KEY

1. (b) 2. (c) 3. (a) 4. (c) 5. (a) 6. (2)

CSIR-UGC-NET Previous Years' Questions

1. A free particle described by a plane wave and moving in the positive z-direction undergoes scattering by a potential $V(r)=\begin{cases}V_0 & \text{if } r\le R\\ 0 & \text{if } r>R\end{cases}$ if V_0 is changes to $2V_0$, keeping R fixed, then the differential scattering cross-section, in the born approximation, **[CSIR June 2012]**

(a) increases to four times the original value (b) increases to twice the original value
(c) decreases to half the original value (d) decreases to one fourth the original value

2. In the Born approximation, the scattering amplitude **f**(θ) for the Yukawa potential $V(r)=\frac{\beta e^{-\mu r}}{r}$ is given by: (in the following $b=2k\sin\frac{\theta}{2}$, $E=\hbar^2k^2/2m$) **[CSIR June 2013]**

(a) $-\frac{2m\beta}{\hbar^2(\mu^2+b^2)^2}$ (b) $-\frac{2m\beta}{\hbar^2(\mu^2+b^2)}$ (c) $-\frac{2m\beta}{\hbar^2\sqrt{\mu^2+b^2}}$ (d) $-\frac{2m\beta}{\hbar^2(\mu^2+b^2)^3}$

3. The scattering amplitude $f(\theta)$ for the Yukawa potential $V(r)=\beta e^{-\mu r}$, where β and μ are positive constants, is given, in the Born approximation by (in the following $b=2k\sin\frac{\theta}{2}$, $E=\hbar^2k^2/2m$)

[CSIR June 2014]

(a) $-\frac{4m\beta\mu}{\hbar^2\left(b^2+\mu^2\right)^2}$ (b) $-\frac{4m\beta\mu}{\hbar^2b^2\left(b^2+\mu^2\right)}$ (c) $-\frac{4m\beta\mu}{\hbar^2\sqrt{b^2+\mu^2}}$ (d) $-\frac{4m\beta\mu}{\hbar^2\left(b^2+\mu^2\right)^3}$

4. The differential cross-section for scattering by a target is given by $\frac{d\sigma(\theta,\varphi)}{d\Omega}=a^2+b^2\cos^2\theta.$ If N is the flux of the incoming particles, the number of particles scattered per unit time is **[CSIR June 2015]**

(a) $\frac{4\pi}{3}N(a^2+b^2)$ (b) $4\pi N\left(a^2+\frac{1}{6}b^2\right)$

(c) $4\pi N\left(\frac{1}{2}a^2+\frac{1}{3}b^2\right)$ (d) $4\pi N\left(a^2+\frac{1}{3}b^2\right)$

5. A particle of energy E scatters off a repulsive spherical potential **[CSIR June 2015]**

$$V(r)=\begin{cases}V_0 & \text{for} \quad r<a\\ 0 & \text{for} \quad r\ge a\end{cases}$$

where V_0 and a are positive constants. In the low energy limit, the total scattering cross-section is $\sigma=4\pi a^2\left(\frac{1}{ka}\tanh ka-1\right)^2$, where $k^2=\frac{2m}{\hbar^2}(V_0-E)>0$. In the limit $V_0\to\infty$ the ratio of σ to the classical scattering cross-section off a sphere of radius a is

(a) 4 (b) 3 (c) 1 (d) 1/2

6. In the scattering of some elementary particles, the scattering cross-section σ is found to depend on the total energy E and the fundamental constants h (Planck's constant) and c (the speed of light in vacuum). Using dimensional analysis, the dependence of σ on these quantities is given by **[CSIR Dec. 2015]**

(a) $\sqrt{\frac{hc}{E}}$ (b) $\frac{hc}{E^{3/2}}$ (c) $\left(\frac{hc}{E}\right)^2$ (d) $\frac{hc}{E}$

7. A particle is scattered by a central potential $V(r)=V_0\, re^{-\mu r}$, where V_0 and μ are positive constants. If the momentum transfer $\vec{q}$ is such that $q=|\vec{q}|\gg\mu$, the scattering cross-section in the Born approximation, as $q\to\infty$, depends on q as **[CSIR Dec. 2016]**

[You may use $\int x^n e^{ax}\,dx=\frac{d^n}{da^n}\int e^{ax}\,dx$]

(a) q^{-8} (b) q^{-2} (c) q^2 (d) q^6

8. After a perfectly elastic collision of two identical balls, one of which was initially at rest, the velocities of both the balls are non-zero. The angle θ between the final velocities (in the lab frame) is **[CSIR Dec. 2016]**

(a) $\theta=\frac{\pi}{2}$ (b) $\theta=\pi$ (c) $0<\theta<\frac{\pi}{2}$ (d) $\frac{\pi}{2}<\theta\le\pi$

9. Consider the potential $V(\vec{r})=\sum_i V_0 a^3\delta^{(3)}(\vec{r}-\vec{r}_i)$, where $\vec{r}_i$ are the position vectors of the vertices of a cube of length a centered at the origin and V_0 is a constant. If $V_0 a^2\ll\frac{\hbar^2}{m}$, the total scattering cross-section, in the low energy limit, is **[CSIR June 2017]**

(a) $16a^2\left(\frac{mV_0a^2}{\hbar^2}\right)$ (b) $\frac{16a^2}{\pi^2}\left(\frac{mV_0a^2}{\hbar^2}\right)^2$ (c) $\frac{64a^2}{\pi}\left(\frac{mV_0a^2}{\hbar^2}\right)^2$ (d) $\frac{64a^2}{\pi^2}\left(\frac{mV_0a^2}{\hbar^2}\right)$

10. A phase shift of 30° is observed when a beam of particles of energy 0.1 MeV is scattered by a target. When the beam energy is changed, the observed phase shift is 60°. Assuming that only s-wave scattering is relevant and that the cross-section does not change with energy, the beam energy is **[CSIR Dec. 2017]**
(a) 0.4 MeV (b) 0.3 MeV (c) 0.2 MeV (d) 0.15 MeV

11. The differential scattering cross section $d\sigma/d\Omega$ for the central potential $V(r)=\frac{\beta}{r}e^{-\mu r}$, where β and μ are positive constants, is calculated in the first Born approximation. Its dependence on the scattering angle θ is proportional to **[CSIR June 2018]**
(A is a constant below).

(a) $\left(A^2+\sin^2\frac{\theta}{2}\right)$ (b) $\left(A^2+\sin^2\frac{\theta}{2}\right)^{-1}$ (c) $\left(A^2+\sin^2\frac{\theta}{2}\right)^{-2}$ (d) $\left(A^2+\sin^2\frac{\theta}{2}\right)^2$

12. In the partial wave expansion, the differential scattering cross-section is given by

$$\frac{d\sigma}{d(\cos\theta)} = \left|\sum_l (2l+1)e^{i\delta_l}\sin\delta_l P_l(\cos\theta)\right|^2$$ **[CSIR June 2019]**

where θ is the scattering angle. For a certain neutron-nucleus scattering, it is found that the two lowest phase shifts δ_0 and δ_1 corresponding to *s*-wave and *p*-wave, respectively, satisfy $\delta_1 \approx \delta_0/2$. Assuming that the other phase shifts are negligibly small, the differential cross-section reaches its minimum for $\cos\theta$ equal to

(a) 0 (b) ±1 (c) $-\frac{2}{3}\cos^2\delta_1$ (d) $\frac{1}{3}\cos^2\delta_1$

13. The elastic scattering of a charged particle of mass *m* off an atom can be approximated by the potential $V(r) = \frac{\alpha}{r}e^{-r/R}$, where α and R are positive constants. If the wave number of the incoming particle is k and the scattering angle is 2θ, the differential cross-section in the Born approximation is **[CSIR June 2019]**

(a) $\dfrac{m^2\alpha^2R^4}{4\hbar^4(1+k^2R^2\sin^2\theta)}$ (b) $\dfrac{m^2\alpha^2R^4}{\hbar^4(2k^2R^2+\sin^2\theta)^2}$

(c) $\dfrac{2m^2\alpha^2R^4}{\hbar^4(2k^2R^2+\sin^2 2\theta)}$ (d) $\dfrac{4m^2\alpha^2R^4}{\hbar^4(1+4k^2R^2\sin^2\theta)^2}$

ANSWER KEY

1. (a) 2. (b) 3. (a) 4. (d) 5. (a) 6. (c) 7. (a)
8. (a) 9. (c) 10. (b) 11. (c) 12. (c) 13. (d)

CSIR-UGC-NET Previous Years' Questions

1. A particle in one dimension moves under the influence of a potential $V(x) = ax^6$, where a is a real constant. For large n the quantized energy level E_n depends on n as: **[CSIR June 2011]**

(a) $E_n \sim n^3$ (b) $E_n \sim n^{4/3}$ (c) $E_n \sim n^{6/5}$ (d) $E_n \sim n^{3/2}$

2. A variational calculation is done with the normalized trial wavefunction $\psi(x) = \frac{\sqrt{15}}{4a^{5/2}}\left(a^2 - x^2\right)$ for the one dimensional potential well $V(x) = \begin{cases} 0 \text{ if } & |x| \le a \\ \infty \text{ if } & |x| > a \end{cases}$

The ground state energy is estimated to be **[CSIR June 2012]**

(a) $\frac{5\hbar^3}{3ma^2}$ (b) $\frac{3\hbar^2}{2ma^2}$ (c) $\frac{3\hbar^2}{5ma^2}$ (d) $\frac{5\hbar^2}{4ma^2}$

3. What would be the ground state energy of the Hamiltonian

$$H = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2} - \alpha\delta(x)$$

if variational principle is used to estimate it with the trial wavefunction $\psi(x) = A\, e^{-bx^2}$ with b as the variational parameter?

[Hint: $\int_{-\infty}^{\infty} x^{2n} e^{-2bx^2} dx = (2b)^{-n-\frac{1}{2}}\Gamma\left(n + \frac{1}{2}\right)$] **[CSIR Dec 2012]**

(a) $-m\alpha^2/2\hbar^2$ (b) $-2m\alpha^2/\pi\hbar^2$ (c) $-m\alpha^2/\pi\hbar^2$ (d) $m\alpha^2/\pi\hbar^2$

4. The bound on the ground state energy of the Hamiltonian with an attractive delta-function potential, namely

$$H = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2} - \alpha\delta(x)$$

using the variational principle with the trial wavefunction $\psi(x) = A\exp(-bx^2)$ is **[CSIR June 2013]**

(a) $-m\alpha^2/4\pi\hbar^2$ (b) $-m\alpha^2/2\pi\hbar^2$ (c) $-m\alpha^2/\pi\hbar^2$ (d) $-m\alpha^2/\sqrt{5}\pi\hbar^2$

5. Consider a particle of mass m in the potential $V(x) = a|x|, a > 0$. The energy eigen-values $E_n (n = 0, 1, 2, ...)$, in the WKB approximation, are **[CSIR Dec 2014]**

(a) $\left[\frac{3a\hbar\pi}{4\sqrt{2m}}\left(n + \frac{1}{2}\right)\right]^{1/3}$ (b) $\left[\frac{3a\hbar\pi}{4\sqrt{2m}}\left(n + \frac{1}{2}\right)\right]^{2/3}$ (c) $\frac{3a\hbar\pi}{4\sqrt{2m}}\left(n + \frac{1}{2}\right)$ (d) $\left[\frac{3a\hbar\pi}{4\sqrt{2m}}\left(n + \frac{1}{2}\right)\right]^{4/3}$

6. The ground state energy of the attractive delta function potential $V(x) = -b\delta(x)$, where $b > 0$, calculated with the variational trial function $\psi(x) = \begin{cases} A\cos\frac{\pi x}{2a}, & \text{for } -a < x < a, \\ 0, & \text{otherwise,} \end{cases}$ is **[CSIR Dec 2014]**

(a) $-\frac{mb^2}{\pi^2\hbar^2}$ (b) $-\frac{2mb^2}{\pi^2\hbar^2}$ (c) $-\frac{mb^2}{2\pi^2\hbar^2}$ (d) $-\frac{mb^2}{4\pi^2\hbar^2}$

7. The ground state energy of a particle in the potential $V(x) = g\,|x|$, estimated using the trial wavefunction

$$\psi(x) = \begin{cases} \sqrt{\dfrac{c}{a^5}}(a^2 - x^2), & x < |a| \\ 0, & x \geq |a| \end{cases}$$ (where g and c are constants) is **[CSIR Dec. 2015]**

(a) $\frac{15}{16}\left(\frac{\hbar^2 g^2}{m}\right)^{1/3}$ (b) $\frac{5}{6}\left(\frac{\hbar^2 g^2}{m}\right)^{1/3}$ (c) $\frac{3}{4}\left(\frac{\hbar^2 g^2}{m}\right)^{1/3}$ (d) $\frac{7}{8}\left(\frac{\hbar^2 g^2}{m}\right)^{1/3}$

8. The energy levels for a particle of mass m in the potential $V(x) = \alpha|x|$, determined in the WKB approximation

$\sqrt{2m}\int_a^b \sqrt{E - V(x)}\,dx = \left(n + \frac{1}{2}\right)\hbar\pi$, where a, b are the turning points and $n = 0, 1, 2, \ldots$), are

(a) $E_n = \left[\frac{\hbar\pi\alpha}{4\sqrt{m}}\left(n + \frac{1}{2}\right)\right]^{2/3}$ (b) $E_n = \left[\frac{3\hbar\pi\alpha}{4\sqrt{2m}}\left(n + \frac{1}{2}\right)\right]^{2/3}$ **[CSIR June 2016]**

(c) $E_n = \left[\frac{3\hbar\pi\alpha}{4\sqrt{m}}\left(n + \frac{1}{2}\right)\right]^{2/3}$ (d) $E_n = \left[\frac{\hbar\pi\alpha}{4\sqrt{2m}}\left(n + \frac{1}{2}\right)\right]^{2/3}$

9. The ground state energy of a particle of mass m in the potential $V(x) = \frac{\hbar^2\beta}{6m}x^4$, estimated using the normalized

trial wavefunction $\psi(x) = \left(\frac{\alpha}{\pi}\right)^{1/4} e^{-\alpha x^2/2}$ is **[CSIR June 2016]**

[Use $\sqrt{\frac{\alpha}{\pi}}\int_{-\infty}^{\infty} dx\, x^2 e^{-\alpha x^2} = \frac{1}{2\alpha}$ and $\sqrt{\frac{\alpha}{\pi}}\int_{-\infty}^{\infty} dx\, x^4 e^{-\alpha x^2} = \frac{3}{4\alpha^2}$].

(a) $\frac{3}{2m}\hbar^2\beta^{1/3}$ (b) $\frac{8}{3m}\hbar^2\beta^{1/3}$ (c) $\frac{2}{3m}\hbar^2\beta^{1/3}$ (d) $\frac{3}{8m}\hbar^2\beta^{1/3}$

10. The Dirac Hamiltonian $H = c\vec{\alpha}\cdot\vec{p} + \beta mc^2$ for a free electron corresponds to the classical relation $E^2 = p^2c^2 + m^2c^4$. The classical energy-momentum relation of a particle of charge q in a electromagnetic potential $(\phi, \vec{A})$ is $(E - q\phi)^2 = c^2\left(\vec{p} - \frac{q}{c}\vec{A}\right)^2 + m^2c^4$. Therefore, the Dirac Hamiltonian for an electron in an electromagnetic field is **[CSIR June 2015]**

(a) $c\vec{\alpha}\cdot\vec{p} + \frac{e}{c}\vec{A}\cdot\vec{A} + \beta mc^2 - e\phi$ (b) $c\vec{\alpha}\cdot\left(\vec{p} + \frac{e}{c}\vec{A}\right) + \beta mc^2 + e\phi$

(c) $c\left(\vec{\alpha}\cdot\vec{p} + e\phi + \frac{e}{c}\left|\vec{A}\right|\right) + \beta mc^2$ (d) $c\vec{\alpha}\cdot\left(\vec{p} + \frac{e}{c}\vec{A}\right) + \beta mc^2 - e\phi$

11. The dynamics of a free relativistic particle of mass m is governed by the Dirac Hamiltonian $H = c\vec{\alpha}.\vec{p} + \beta mc^2$, where $\vec{p}$ is the momentum operator and $\vec{\alpha} = (\alpha_x, \alpha_y, \alpha_z)$ and β are four 4×4 Dirac matrices. The acceleration operator can be expressed as **[CSIR Dec. 2016]**

(a) $\frac{2ic}{\hbar}(c\vec{p} - \vec{\alpha}H)$ (b) $2ic^2\vec{\alpha}\beta$ (c) $\frac{ic}{\hbar}H\vec{\alpha}$ (d) $-\frac{2ic}{\hbar}(c\vec{p} + \vec{\alpha}H)$

12. Using the trial function $\psi(x) = \begin{cases} A(a^2 - x^2) & ; \ -a < x < a \\ 0 & ; \ \text{otherwise} \end{cases}$, the ground state energy of a one-dimensional harmonic oscillattor is **[CSIR June 2017]**

(a) $\hbar\omega$ (b) $\sqrt{\frac{5}{14}}\hbar\omega$ (c) $\frac{1}{2}\hbar\omega$ (d) $\sqrt{\frac{5}{7}}\hbar\omega$

13. The energy eigenvalues E_n of a quantum system in the potential $V = cx^6$ (where $c > 0$ is a constant), for large values of the quantum number n, varies as **[CSIR Dec. 2017]**

(a) $n^{4/3}$ (b) $n^{3/2}$ (c) $n^{5/4}$ (d) $n^{6/5}$

14. The n-th energy eigenvalue E_n of a one-dimensional Hamiltonian $H = \frac{p^2}{2m} + \lambda x^4$, (where $\lambda > 0$ is a constant) in the WKB approximation, is proportional to **[CSIR June 2018]**

(a) $\left(n+\frac{1}{2}\right)^{4/3}\lambda^{1/3}$ (b) $\left(n+\frac{1}{2}\right)^{4/3}\lambda^{2/3}$ (c) $\left(n+\frac{1}{2}\right)^{5/3}\lambda^{1/3}$ (d) $\left(n+\frac{1}{2}\right)^{5/3}\lambda^{2/3}$

15. A one-dimensional system is described by the Hamiltonian $H = \frac{p^2}{2m} + \lambda|x|$, where $\lambda > 0$. The ground state energy varies as a function of λ as **[CSIR Dec. 2018]**

(a) $\lambda^{5/3}$ (b) $\lambda^{2/3}$ (c) $\lambda^{4/3}$ (d) $\lambda^{1/3}$

16. The Hamiltonian of a quantum particle of mass m is $H = \frac{p^2}{2m} + \alpha\,|\,x\,|^r$, where α and r are positive constants. The energy E_n of the n^{th} level, for large n, depends on n as **[CSIR June 2019]**

(a) n^{2r} (b) n^{r+2} (c) $n^{1/(r+2)}$ (d) $n^{2r/(r+2)}$

ANSWER KEY

1. (d) **2. (d)** **3. (c)** **4. (c)** **5. (b)** **6. (b)** **7. (a)**
8. (b) **9. (d)** **10. (d)** **11. (a)** **12. (b)** **13. (b)** **14. (a)**
15. (b) **16. (d)**

JEST & TIFR Previous Years' Questions

1. Consider a particle confined by a potential $V(x) = k|x|$, where k is a positive constant. The spectrum E_n of the system, within the WKB approximation, is proportional to **[JEST 2017]**

(a) $\left(n+\frac{1}{2}\right)^{3/2}$ (b) $\left(n+\frac{1}{2}\right)^{2/3}$ (c) $\left(n+\frac{1}{2}\right)^{1/2}$ (c) $\left(n+\frac{1}{2}\right)^{4/3}$

2. Given a particle confined in a one-dimensional box between $x = -a$ and $x = +a$, a student attempts to find the ground state by assuming a wave-function **[TIFR 2018]**

$$\psi(x) = \begin{cases} A(a^2 - x^2)^{3/2} & \text{for } |x| \le a \\ 0 & \text{for } |x| > a \end{cases}$$

The ground state energy E_m is estimated by calculating the expectation value of energy with this trial wave-function. If E_0 is the true ground state energy, what is the ratio E_m/E_0 ?

ANSWER KEY

1. (b) **2. (1.065)**